Essays in Labor and Product Market Search

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This thesis studies markets where trades between buyers and sellers are impaired by meeting frictions such as imperfect information (buyers need to search for sellers), or coordination problems (workers do not observe to which jobs other workers apply). I model meeting frictions in a new and very general way and show how they interact with the optimal selling mechanism. The thesis offers an explanation why in some markets we observe that sellers use auctions while in other markets we observe price posting. Despite of frictions and asymmetric information, decentralized markets can under certain conditions deliver social efficient outcomes.

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Contents

CHAPTER 1

Introduction

This thesis studies markets with frictions. A market without frictions, a competitive market, is defined by a commodity and a set of sellers and buyers. Examples of commodities are not only bread, coffee beans, or computers, but also labor services of economists. The absence of frictions means that at each instant of time, buyers can instantly get the commodity at some publicly known price. All that matters in this market is the publicly known equilibrium price and at this equilibrium price supply equals demand.

At first glance, it seems that this concept approximates many real life examples very closely. For example, if you want to buy bread, you can go to a local bakery. You know more or less the price and you can buy as much bread as you want. Of course, you can not get the bread instantaneously; you need to visit a store. But the time cost is small enough that the local bread market approximates reasonably well a competitive market. However, in an influential paper, Diamond (1971) shows that even with a very small time cost, in a sequential search market each seller will charge the monopoly price. The reason is that buyers in the sequential search market can meet at most one seller per unit of time, and it takes some time for the buyer to meet the next seller. Because of this time cost, if some price is acceptable for the buyer, a price slightly higher will also be acceptable as long as the new price is below the buyer's valuation. In the end, each seller will increase its price until there is no room for further increase, i.e., the equilibrium price will be the monopoly price.

Diamond (1971) shows, in the most stark way, that for a clear understanding of the market outcome it matters how buyers and sellers meet and what the protocols of trade are. Seemingly small deviations from the competitive market model can thus have a huge impact on market outcomes.

In an extension of the Diamond (1971) model, Burdett and Judd (1983) show that if a buyer can observe prices of two or more stores at the same time, instead of posting the monopoly price, sellers will use a mixed strategy: they post different prices. First, no seller will post a price equal to the production cost, because even when the seller would charge the monopoly price, this does not rule out a transaction (when he has a buyer who has visited no other sellers). This will result in a positive profit for the seller. Also, if there exists a mass point in the market price distribution, then a small downward deviation by sellers in that group will reduce the selling price only marginally, but it will increase the selling probability substantially. Therefore, therecan not be mass points in the market price distribution. Both Diamond (1971) and Burdett and Judd (1983) consider one special protocol of trade: price posting (sellers post and commit to the posted price). In Burdett and Judd (1971), pure price dispersion occurs in general, but the probability that a buyer meets n buyers, $n = 0, 1, 2, \ldots$, is left unspecified.

In this thesis, a market always has two sides, i.e., workers and firms or sellers and buyers. Sometimes the role of buyers and sellers are reversed. To avoid confusion, the following terminology is introduced. Denote the two sides of the market as A and B. In this thesis, a meeting is always n-to-1, i.e., n agents from side A and 1 agent from side B. In Burdett and Judd (1983), the n-side consists of the sellers. In Chapter 2 and 4, the n-side consists of the firms; in Chapter 3, the n-side consists of the buyers; in Chapter 5, the n-side are the sellers.

In Chapter 3, the analysis of Burdett and Judd (1983) is extended. The probability that a seller meets n buyers is determined by market tightness, and it is referred to as the meeting technology. A new function $\phi(\mu, \lambda)$, the probability that a seller meets at least one buyer from a given subgroup, is introduced, where μ is the relative measure of the subgroup and λ is the relative measure of all buyers. The nice thing about this new function ϕ is that it is an alternative representation for the meeting technology and that it makes the analysis of this market tractable. In Chapter 3, sellers can post any selling mechanism. Because of frictions, a buyer can only visit one seller at a time. However, buyers can observe all posted mechanisms before deciding which seller to visit. If multiple buyers show up at a seller, then the selling mechanism will decide which buyer wins. The key questions are: Which mechanisms will be posted? What will be the equilibrium distribution of mechanisms? How do sellers and buyers sort into different submarkets? The problem seems to be intractable because there are so many mechanisms a seller can choose from. However, sellers must compete with each other in attracting buyers, and a buyer will visit some seller only if the seller is among the best choices the buyer has. Meanwhile, if a seller wants to attract buyers, the seller will offer the buyer the highest value that the buyer can get somewhere else, but not more than that. In equilibrium, there will exist a "market utility" for each buyer type. If a seller offers less that the market utility for a particular buyer type, she will not get any buyers from that type. Since sellers are the residual claimants, they will only post efficient mechanisms. But this does not mean that all sellers will post the same efficient mechanism. In general, sellers can not do better than posting a second-price auction with an entry fee. An auction ensures that in an n-to-1 meeting, the buyer with the highest valuation wins. The entry fee internalizes the externalities buyers impose on each other which can be easily calculated in terms of ϕ. By varying the entry fee, sellers can attract different queues. In equilibrium, sellers use the above mechanisms to "buy" queues where prices equal the market utility function. Two extreme cases of market segmentation are considered: pooling and complete segmentation. Pooling means that all sellers post the same (equivalent) mechanism and all sellers and buyers search in one market. Complete segmentation means that sellers will post different mechanisms for different types of buyers, and different types of buyers search in different submarkets.

In Chapter 4, the n-side consists of firms and the 1-side of workers. As in Chapter 2 and 5, the n-side can not observe the terms of trade ex ante, and needs to search for an agent from the 1-side in a purely random way. So search is random instead of directed. This chapter considers the efficiency of vacancy creation (or the optimality of market tightness) in the decentralized equilibrium. In the model, there is a fixed measure of workers but firms can enter the market freely. At the beginning of each period, each firm observes a private signal indicating its productivity for the period and needs to decide whether to enter the market or not. Workers are assumed not to know the productivity of a potential job (incomplete information). To focus on the effects of search frictions on social efficiency, two assumptions are made: (i) unemployed and employed workers search equally efficiently, (ii) wage mechanisms are efficient, i.e., the firm with the highest productivity in a multilateral meeting always wins, and firms whose productivity equals the workers' leisure value have value zero. Examples of efficient mechanisms are: (i) wage posting and (ii) a second-price auction without an entry fee. By the celebrated revenue equivalence result in the mechanism design literature, all wage mechanisms satisfying the above two requirement are payoff equivalent for all workers and firms. By using the new function ϕ of Chapter 3, the problem becomes very tractable and the answer to the entry efficiency question very intuitive. If an additional firm reduces the meeting probabilities of existing firms, then firms impose negative externalities on each other. Because the social contribution of a firm is smaller than its own expected value, in equilibrium there will be excessive firm entry (vacancy creation). The opposite holds when an additional firm entering increases the meeting probabilities of existing firms. Unlike in the model in Chapter 3, with common wage mechanisms like wage posting, the externalities are not internalized.

Chapters 2 and 5 study policy questions. Chapter 2 analyzes how collective wage bargaining affects social efficiency. In many European countries, collective wage bargaining restricts the set of possible wages. Chapter 2 focuses on the following tradeoff. Under CBA, wage dispersion is reduced so that a high and a low productivity firm may have to offer the same wage. Thus even if a worker who is employed at the low productivity firm gets poached by the high productivity firm, the worker might not move and therefore the average productivity in the market will be lower. On the other hand, in the decentralized equilibrium without CBA, firms do not internalize search and business stealing externalities they impose on each other, and therefore too many vacancies can be created. The net effect of CBA depends on the relative weight of the above tradeoffs. The model in Chapter 2 can be thought as a continuous-time variant of the model in Chapter 4 with two differences: (i) employed workers search less efficiently than unemployed workers, (ii) the value of a potential match is known to both the worker and the firm (complete information) and the wage is determined by a bilateral bargaining with the worker's outside option being unemployment and the firm's outside option being equal to the value of a vacancy (which is zero in equilibrium).

Chapter 5 studies the effects of a statutory minimum price on the market price distribution. The model is a version of the one in Chapter 4, where the price mechanism is assumed to be price posting. Countries usually apply statutory minimum prices to gasoline, books, diary products and so on. Another example is agricultural products stamped by "Fair Trade". Importers of such products have to pay at least the minimum prices stipulated by Fairtrade International. With a statutory minimum price, the pure price dispersion equilibrium introduced by Burdett and Judd (1983) is no longer viable. There will be a mass point at the minimum price, and if the minimum price is sufficiently high, then all sellers will post it. Furthermore, because of the efficiency result in Chapter 4, the welfare implication of a minimum price can be answered in a simple way. First, because of the mass point at the minimum price, the allocation of goods is not always efficient. This is similar to the effects of CBA on job-to-job movements in Chapter 2. However, if the meeting technology is such that new sellers will increase the meeting probabilities of existing sellers (and thus equilibrium entry in the absence of a minimum price is too low), then a statutory minimum price will increase the average price in the market and thus generate more entry. The overall welfare implications depend on the relative importance of the above two effects.

CHAPTER 2

Collective versus decentralized wage bargaining and the efficient allocation of resources[1]

2.1 Introduction

Under collective bargaining agreements (CBA), wages are determined at the industry level rather than at the individual level.[2] An advantage of collective wage bargaining is that potential externalities can be internalized while a disadvantage is that the allocative role of wages is reduced which leads to a sub optimal allocation of workers over jobs. To study this trade-off we need a model that allows for two sided heterogeneity and search frictions. Heterogeneity is important because we are interested in allocation. Search frictions are important because it takes time for workers to find the production units where they are needed most. Wages play in this setting a key role because they inform workers which firms need them most.

[1] This chapter is based on Cai et al. (2014)

[2] Even if unionization rates are low, CBA coverage can be substantial. In the Netherlands, the minister can for example make a CBA binding for an entire sector.

There exists a lot of cross-country variation in coverage rates of CBA, see OECD (2004, 2012). In the US, the fraction of workers who are covered by a collective agreement has been falling from 26% in 1980 to 13% in 2010. In the Netherlands, this fraction has been increasing over time from 70% in 1980 to over 80% in 2012. In Germany the CBA coverage rate fell from 80% in 1980 to 60% in 2010. The relation between coverage rates and wage dispersion has also been studied extensively. Card (1996) and DiNardo et al. (1996) give empirical evidence that unions compress wages and Blau and Kahn (1999), Hartog et al. (2002) and OECD (2004) show that in countries where coverage is high there is less wage dispersion.

To study the relation between CBA, wage dispersion and the allocation of workers we use a model similar to Marimon and Zilibotti (1999) and Gautier et al. (2010). The idea is that worker (s) and job types (c) are located on a circle and productivity is decreasing in the distance, x, between s and c.[3] In the simplest version of the centralized-wage setting case, all firm types offer the socially optimal wage under the constraint that it is the same for all job types while in the decentralized case, firm types are allowed to post different wages to different worker types. We assume that firms *cannot* ex ante commit to a wage schedule (if firms can commit, the decentralized outcome is more favorable).[4] The reason that firms pay positive wages that are increasing in productivity, even if they have all the bargaining power is that a higher wage reduces the quit rate.

Gautier et al. (2010) show that in this case firms engage in excessive vacancy creation due to a *business-stealing effect*. The idea is that firms do not internalize the output loss of firms they poach a worker from, in particular, they do not care whether they destroy relatively good or bad matches. Although each worker's transition is efficient, the expected marginal increase in aggregate output is too low to justify the entry cost. In the simplest CBA case where the union can set only one wage, the wage can be set at a level that generates the efficient level of entry. However, if all firms in a sector pay the same wage, workers do not know which firms need their services most and once they have

[3]The results of Gautier et al. (2006) show that without on-the-job search, the circular model has the same characteristics as a Taylor expansion of the hierarchical model.

[4]Under continuous renegotiation, ex ante commitment becomes meaningless.

a job they stop searching. We are interested in the trade-off between efficient entry and efficient allocation and which wage mechanism is most desirable from a social welfare point of view.

The desirability of CBA mainly depends on the efficiency of on-the-job search relative to off the job search (ψ). If employed job seekers receive more than 20% of the number of offers as the unemployed workers (for the commitment case it is more than 10%), the cost of CBA exceed the benefits. The reason for this is that the more efficient on-the-job search is, the faster workers flow to the jobs where they are needed most and the more costly it is if CBA prevents firms within an industry to pay a higher wage if a particular worker type is very valuable for them. One novelty relatively to Gautier et al. (2010) is that we also solve the model for general bargaining power. This allows us to derive a relationship between the desirability of CBA and a worker's bargaining power. For the estimated values of ψ, the decentralized case performs better for most values of β. For intermediate values of β, the threshold value of ψ above which decentralized wage setting performs better goes down because the business-stealing-externality is reduced. For very high values of β (above 0.8), the standard congestion externalities become more important and too few vacancies are created under decentralized bargaining.

In most countries, CBA takes the form of pay scales. Therefore, in section 2.5, we allow the union to choose upon n different wages. Given this set of wages, a firm decides after observing the quality of the match, which of the n wages it offers to the worker. To our knowledge, this is the first paper that endogenizes union-based pay scales. We find that $n = 4$ performs almost as well as $n \to \infty$. CBA with sufficiently many pay scales ($n \geq 4$) is only socially more efficient for very low β's (≤ 0.1) and for very high β's (≥ 0.7) than the decentralized wage setting scheme. The main reason for this is that they set the lowest wage in the pay scale too high. If pay scales contain to few wages, there are again not enough transitions from bad to good matches.

There are a number of other papers that study the effect of centralized bargaining in frictional labor markets. Lindbeck and Snower (1986) consider insider-wage setting. Pissarides (1986) asks whether the standard search externalities will be internalized by a union. He finds that this is the case only if the union's policy is chosen by unemployed persons. If employed persons can influence the union's policy, unemployment and wages will be too high. We find that in a dynamic setting with modest discount rates, it matters very little whether unions maximize the expected welfare of the average employed worker or of unemployed workers. In both cases, the union realizes that at some point in time, employed workers may become unemployed and that too high wage demands are welfare reducing because it reduces vacancy creation. Therefore, in a dynamic setting, the negative welfare effects of insider-wage setting are a lot smaller than in a static model. Other models that have frictions and centralized bargaining include Delacroix (2006) and Bauer and Lingens (2014). None of those models look at the trade-off between an efficient allocation and internalizing externalities as we do. Teulings and Hartog (1998) argue that an advantage of CBA is that it reduces hold-up problems (because individual workers and firms cannot influence the wage) while at the same time it also allows wages to respond to aggregate shocks. This chapter ignores this and only looks at wage dispersion across jobs. Finally, Krusell and Rudanko (2012) focus on union wage setting rather than CBA's. They assume that in the short run, unions raise current wages above the efficient level, in order to appropriate surpluses from firms with existing matches. We abstract from that here but since this makes the CBA perform worse, it will not change our main conclusions about the performance of decentralized wage setting and CBA.

The chapter is organized as follows. Section 2.2 starts with the assumptions, derives the equilibrium conditions, and characterizes the equilibrium. Section 2.3 discusses the two wage mechanisms. Section 2.4 conducts welfare analysis and section 2.5 introduces pay scales and unions maximizing the value of employment rather than unemployment. Finally, section 2.6 concludes.

2.2 The model

Assumptions

For the decentralized case, we use the model of Gautier et al. (2010) which extends Marimon and Zilibotti (1999) to allow for on-the-job search. The model is briefly summarized below. Worker types (s) and job types (c) are locations on a circle with circumference 1. The production technology has constant-returns-to scale so it is easiest to think of firms as consisting of one worker. A matched firm-worker pair produces Y which depends on the "spherical distance" between s and c: $x(s,c) = \min\{|s-c|, 1-|s-c|\}$, which is common knowledge to both the worker and the firm. Note $0 \leq x(s,c) \leq 1/2$. Specifically, $Y(s,c) = Y(x)$. Since we interpret x as an indicator of mismatch between workers and jobs, $Y(x)$ has a maximum at 0, and the value of the maximum is normalized to unity: $Y(0) = 1$. We assume that $Y(x)$ is twice differentiable and strictly quasiconcave. If we think of $Y(x)$ as a second order Taylor approximation of a more general (differentiable) production function, around the optimal assignment, the derivative of $Y(x)$ at 0 should be 0. The simplest functional form that meets those criteria is,

$$Y(x) = 1 - \frac{1}{2}\gamma x^2.$$

Low values of γ imply that a precise match is not very important. In the limit, $\gamma \to 0$, the model reduces to a standard Diamond-Mortensen-Pissarides type of matching model with identical workers and firms.

We assume that both labor and vacancy supply are uniformly distributed over the circle (the latter can be shown to be an equilibrium). Total labor supply in period t equals $L(t)$ and the total number of vacancies per unit of labor supply is given by $v(c) = v$. The flow cost of maintaining a vacancy is equal to K per period and the flow value of non-market time is B.

We assume that the discount rate ρ equals the population growth rate (golden growth) and that all new workers start out as unemployed. The implications of this assumption are the same as when we assume that the discount rate ρ is much smaller than the job-finding and separation rate, $\rho \ll \delta, \lambda$. This is a common assumption in the wage-posting literature (see for example, Burdett and Mortensen (1998)).

Next, we discuss the job search technology. Let m be the total number of contacts between job seekers and vacancies per unit of labor supply and u be the unemployment rate. We think it is reasonable to assume that two workers with an empty intersection of matching sets do not cause congestion on each other. Therefore, we take a quadratic contact technology,

$$m = \lambda \left[u + \psi (1-u) \right] v$$

The parameter $\psi, 0 \leq \psi \leq 1$, measures the relative efficiency of on-the-job search versus search while unemployed. Marimon and Zilibotti (1999) consider the case, $\psi = 0$, which is related to the stochastic matching model of Pissarides (2000). If off- and on-the-job search are equally efficient, $\psi = 1$, the model is relatively simple and analytical results can be obtained. For the general ψ case we rely on numerical simulations. The overall efficiency of the matching process is captured by λ. The Walrasian equilibrium is obtained for $\lambda \to \infty$. Finally, matches between workers and jobs are destroyed at an exogenous rate $\delta > 0$.

We focus on two wage-setting schemes. First, we add CBA to this framework. The simplest implementation is to interpret CBA as the constraint that all jobs of the same type must pay the same wage. Section 2.5 relaxes this assumption and allows for pay scales (a union offers firms to choose a wage from a menu with N wages). To make the simplest version of CBA as favorable as possible, we set this wage optimally from a social point of view. In the decentralized case, we assume that firms *cannot* commit on future wage payments and wages are determined by bilaterally efficient bargaining between the worker and the firm, see Coles (2001), Shimer (2006), Gautier et al. (2010) and Coles and Mortensen (2011). So, firms pay only "no-quit" premia but no "hiring" premia. The reason that it is in a firm's interest to pay a positive wage is the same as in efficiency wage models where shirking reduces if the wage goes up. Here, given the behavior of other firms, a higher wage reduces the quit probability and this is more valuable the smaller mismatch, x is. Gautier et al. (2010) also assume that firms *can* commit on a future wage policy that is contingent on x in the current job.[5] In that case, firms pay both "no-quit" premia and "hiring" premia and the business-stealing externality is fully internalized for sufficiently large ψ.[6] Since social welfare in the decentralized outcome is higher with commitment, we can view the no-commitment outcome as a lower bound on welfare in the decentralized case.

[5] In Burdett and Mortensen (1998) firms can also ex ante commit to wages.

[6] We find that under commitment with $\beta = 0$ that for $\psi \geq 0.3$, the market and Planner's outcome are almost identical.

Job dynamics

As long as wages are decreasing in x, workers will move in the direction of their optimal job type. We will first solve for the worker flows and the asset values of vacancies, employment and unemployment for a given wage setting rule assuming that wages are decreasing in x, as is proved in Gautier et al. (2010). This allows us to write the wage schedule as a function of x, $W(x)$ rather than $W(s,c)$. Denote by $G(x)$ the fraction of workers employed at jobs at smaller distance from their optimal job than x.[7] Let $\bar{x}$ be the reservation distance from their optimal job for an unemployed worker. Jobs located at a further distance are declined. This implies that $G(\bar{x}) = 1$. At the golden-growth path, unemployment and employment must grow at the same rate. The inflow into the class of employed workers at distance x or less from their favorite job minus the outflow from this class must therefore equal the population growth rate:

$$2\lambda v x \left\{u + \psi(1-u)\left[1 - G(x)\right]\right\} - \delta(1-u)G(x) = \rho(1-u)G(x). \tag{2.1}$$

The first term on the left-hand side is the number of people that finds a job at a distance less than x from the optimal job. The previous state was either unemployment (the first term in parentheses), or employment at a greater distance than x (the second term in parentheses). The number of better jobs is given by $2vx$, since the worker accepts jobs both to the left and to the right of the optimal job type $s = c$ (or, equivalently, $x = 0$). The second term in brackets is weighted by the factor ψ, reflecting the efficiency of on-the-job search relative to search while unemployed. The final term on the left-hand side is the outflow of workers from the class of matches with mismatch indicator of x or less: $\delta G(x)$. The right-hand side reflects that at the balanced growth path, employment grows at a rate ρ at all levels including the class of workers with a mismatch indicator smaller than x, $G(x)$. Steady-state unemployment can be derived from evaluating (2.1) at $\bar{x}$.

$$u = \frac{1}{1 + \kappa v \bar{x}}, \tag{2.2}$$

$$\text{where } \kappa \equiv \frac{2\lambda}{\rho + \delta}. \tag{2.3}$$

[7] We assume that at a tie, the worker moves with a positive probability; see also Shimer (2006).

Note that on the balanced-growth path, u and $G(x)$ are constant. Substitution of (2.2) for u in the balanced-growth condition (2.1) yields

$$G(x) = 1 - \frac{\bar{x} - x}{\left(1 + \psi\kappa\upsilon x\right)\bar{x}}. \tag{2.4}$$

with the corresponding density

$$g(x) = \frac{1 + \psi\kappa\upsilon\bar{x}}{\bar{x}\left(1 + \psi\kappa\upsilon x\right)^2}. \tag{2.5}$$

Value functions

Let $W(x)$ be the wage for a job with mismatch indicator x and let V^U be the asset value of unemployment. We can write the asset value for a worker holding a job at distance x from his optimal job type, $V^E(x)$ as

$$\rho V^E(x) = W(x) + 2\psi\lambda\upsilon \int_0^x \left[V^E(z) - V^E(x)\right] dz - \delta\left[V^E(x) - V^U\right]. \tag{2.6}$$

Gautier et al. (2010) show that we can rewrite this as (see appendix 2.6 for a derivation),

$$V^E(x) = \frac{W(x)}{\rho + \delta + 2\psi\lambda\upsilon S x} + \frac{\delta}{\rho + \delta} V^U + 2\psi\lambda\upsilon S \int_0^x \frac{W(z)}{\left(\rho + \delta + 2\psi\lambda\upsilon S z\right)^2} dz. \tag{2.7}$$

The asset value for an unemployed job seeker satisfies the following Bellman equation,

$$\rho V^U = B + 2\lambda\upsilon \int_0^{\bar{x}} \left[V^E(x) - V^U\right] dx. \tag{2.8}$$

Evaluating (2.6) at $\bar{x}$ and using the definition of g in (2.5) gives

$$\rho V^E(\bar{x}) = \rho V^U = \frac{W(\bar{x}) + \psi\kappa\upsilon\bar{x}\mathrm{E}_G W}{1 + \psi\kappa\upsilon\bar{x}} = \frac{uW(\overline{x}) + \psi(1-u)\mathrm{E}_G W}{u + \psi(1-u)}. \tag{2.9}$$

The asset value in the marginal job is a weighted average of the wage in the current job $W(\bar{x})$ (the reservation wage) and the expected wage in the next job, $\mathrm{E}_G W$. Finally, we can also write the value of unemployment as a weighted average of the value of leisure B and the expected wage[8],

$$\rho V^U = \frac{B + \kappa\upsilon\bar{x}\mathrm{E}_G W}{1 + \kappa\upsilon\bar{x}} = uB + (1-u)\mathrm{E}_G W, \tag{2.10}$$

where the final step uses (2.2).

[8]Use the fact that the right-hand side of (2.6) and (2.9) are equal allows us to obtain an expression for $\int_0^{\bar{x}} \left[V^E(x) - V^U\right] dx$. Substitution of this expression into (2.8) allows us to write ρV^U as a function of $W(\bar{x})$, which can be substituted out by solving (2.9) for $W(\bar{x})$.

Free entry implies that the option value of a vacancy of type c must be equal to K. Define $\mathrm{E}_G Y \equiv \int_0^{\bar{x}} g(x)\, Y(x)\, dx$ and $\mathrm{E}_G W \equiv \int_0^{\bar{x}} g(x)\, W(x)\, dx$, then,

$$\begin{aligned} K &= 2\lambda \int_0^{\bar{x}} \{u + \psi(1-u)[1-G(x)]\} \frac{Y(x) - W(x)}{\rho + \delta + 2\psi\lambda v x} dx \\ &= \frac{1-u}{v}(\mathrm{E}_G Y - \mathrm{E}_G W). \end{aligned} \tag{2.11}$$

Starting off with the first equality, the first factor in the integrand is the effective fraction of individuals $u + \psi(1-u)[1-G(x)]$ willing to accept a type x match. It equals the number of unemployed, u plus the number of workers employed in jobs with a greater mismatch indicator than x, $(1-u)[1-G(x)]$; the latter number is scaled down with ψ to account for a lower effectiveness of on-the-job search. The second factor is the value of a filled vacancy. Just as in the wage equation, we discount current revenue $Y(x) - W(x)$ by the discount rate ρ plus the separation rate δ plus the quit rate $2\psi\lambda v x$. The second line follows from substitution of the relations for employment and unemployment. Multiplying (2.11) by v gives a simple interpretation of this zero-profit condition: at any point in time, the cost of vacancies, vK, is equal to the employment rate $(1-u)$ times the expected profit in a filled vacancy, $(\mathrm{E}_G Y - \mathrm{E}_G W)$.

Equations (2.2), (2.5), (2.11), (2.9), and (2.10) determine u, v, ρV^U, $\bar{x}$ and $G(x)$.

2.3 Decentralized wage setting without commitment

Combining Equations (2.2) and (2.9), gives

$$W(\bar{x}) = B[u + \psi(1-u)] + (1-\psi)(1-u)E_G(W), \tag{2.12}$$

The wage function $W(x)$ (which we need to derive $\mathrm{E}_G W$) for the general case that $0 < \beta < 1$ follows from

$$W(x) = \arg\max_W \left[\hat{V}^E(W) - V^U\right]^\beta \left[\frac{Y(x) - W}{\rho + \delta + 2\psi\lambda v \bar{x} \widehat{F}(W)}\right]^{1-\beta},$$

where $\widehat{F}(W)$ is the fraction of offered wages $W(x)$ that is equal to or higher than W, and $\hat{V}^E$ means that it is now written as a function of W rather than x. For the special case of $\beta = 0$, which was studied in Gautier et al. (2010), wages are implicitly determined by the following differential equation in x (which follows from maximizing the appropriately discounted value of a job for a firm),

$$W_x(x) = -\frac{\psi\kappa v\left[Y(x) - W(x)\right]}{1+\psi\kappa v x}, \tag{2.13}$$

and the initial condition is given by the following equation,

$$W(\bar{x}) = Y(\bar{x}) = 1 - \frac{1}{2}\gamma\bar{x}^2. \tag{2.14}$$

In Appendix 2.6, we consider the case for general β.

Definition 1. *The equilibrium consists of the set* $\{u, v, \bar{x}, W(\bar{x}), W(x), G(x)\}$ *satisfying the equations (2.2), (2.4),(2.12), (2.13), and (2.14).*

The solution of the differential equation (2.13) is,

$$W(x) = 1 + \gamma\frac{x-\bar{x}}{\psi\kappa v} + \frac{1}{2}\gamma x(x-2\bar{x}) - \gamma\frac{1+\psi\kappa v x}{(\psi\kappa v)^2}\log\left(\frac{1+\psi\kappa v x}{1+\psi\kappa v\bar{x}}\right).$$

It is easy to show that wages are indeed decreasing in mismatch, x. Therefore, wage dispersion is a desirable feature of equilibrium because it motivates workers to move from less to more productive jobs. [9] The difference with Burdett and Mortensen (1998) is that in their setting, firms pay both a hiring and a no-quit premium to workers. In this setting, the lack of commitment makes it impossible to pay hiring premia but firms still pay no-quit premia because it is in their own interest to do so.

Centralized wage setting (CBA)

In this subsection, we derive the equilibrium if all firms are obliged to pay a wage that is determined by collective wage bargaining, W^{cb}. In that case, workers have no incentives to continue searching on the job and $\bar{x}$ follows

[9] If search would be directed, then workers would only apply to an $x = 0$ job and it would be hard to explain mismatch. Here, workers learn about a limited number of wages but over time, they move towards more productive jobs as long as their job is not destroyed.

from the fact that at the marginal job, the worker gets all the output, $Y(\bar{x}) = 1 - \frac{1}{2}\gamma\bar{x}^2 = W^{cb}$, or

$$\bar{x} = \sqrt{\frac{2\left(1 - W^{cb}\right)}{\gamma}}. \tag{2.15}$$

The value of employment is given by,

$$\rho V^E(W^{cb}) = W^{cb} - \delta\left[V^E(W^{cb}) - V^U\right]. \tag{2.16}$$

The asset value for an unemployed job seeker is,

$$\rho V^U = B + 2\lambda\bar{x}v\left[V^E(W^{cb}) - V^U\right], \tag{2.17}$$

where $2\lambda\bar{x}v$ is the set of acceptable jobs.

By using the free entry condition, we can find the values of K for which an equilibrium exists with a positive vacancy stock,

$$v = \frac{1-u}{K}\left(\mathrm{E}_{\tilde{G}}Y - W^{cb}\right). \tag{2.18}$$

There is no on-the-job search in this environment, and consequently there are no job-to-job movements. So the new distribution for mismatch, x, is,

$$\tilde{G}(x) = \frac{x}{\bar{x}}.$$

This is a special case of Equation (2.4) with the on-the-job search parameter ψ equal to 0.

Proposition 2. *Given a collective wage W^{cb}, the equilibrium with CBA is characterized by the following first order condition,*

$$\frac{K\sqrt{\gamma}}{\kappa} = \frac{2\sqrt{2}}{3}u(1 - W^{cb})^{\frac{3}{2}}, \tag{2.19}$$

where κ is given by Equation 2.3.

Proof. Combine Equations (2.2), (2.15), and (2.18) □

From the above equation, it can be easily seen that the unemployment rate u is increasing in W^{cb} since here W^{cb} is still constant. A higher collective wage will lead to a higher unemployment rate.

Efficiency

Define net output Ω as,

$$\begin{aligned}\Omega &\equiv (1-u)\,\mathrm{E}_G Y + uB - vK \\ &= (1-u)\,\mathrm{E}_G Y + uB - (1-u)\,(\mathrm{E}_G Y - \mathrm{E}_G W) \\ &= uB + (1-u)\,\mathrm{E}_G W.\end{aligned} \tag{2.20}$$

Note the equality in the second line holds in both the decentralized wage bargaining and the collective wage bargaining. The first term is actual output of the employed workers, the second term captures the value of leisure of the unemployed job seekers, and the third term is the cost of keeping vacancies open. Under free entry the difference between output and wages is spent on vacancy creation and the final step tells us that net output is equal to the value of unemployment, see (2.10).[10]

So far we haven't specified any rule for wage determination in the CBA case. In the baseline analysis, we assume that the wage in the CBA case is set to maximize the value of unemployment. By combining (2.19) and (2.20), the wage that maximizes the asset value of unemployment satisfies,

$$K(1-\frac{3}{2}B) = \kappa\sqrt{\frac{2}{\gamma}}(1-W^{cb})^{\frac{3}{2}} - \frac{1}{2}KW^{cb}.$$

In the next section, we will solve the model numerically for general β and ψ and derive the conditions under which CBA is more desirable than decentralized wage setting. In section 2.5 we allow the union to define pay scales and maximize the value of employment rather than unemployment.

2.4 Calibration and numerical simulations

The irrelevance of the scale of mismatch

As Gautier and Teulings (2015) show, for the equilibrium only $\kappa = 2\lambda/(\rho+\delta)$ matters and not the individual values for λ, ρ, or δ. For the decentralized case, this can be seen from the equations determining the equilibrium in Definition

[10]Note this holds also in the CBA case, which can be shown by combining Equations (2.2), (2.16) and (2.17).

1. We extend their analysis to allow for general β. The derivation can be found in Appendix 2.6. In the CBA case, the equilibrium has a similar property. This can be seen from Equation (2.19) which determines the trade-off between the unemployment rate and the centralized optimal wage. Furthermore, only the value of the composite variable, $K\sqrt{\gamma}/\kappa$ matters for equilibrium not the individual values for κ, γ, or K. If we increase K to $2K$ and κ to 2κ, keeping ρ and δ unchanged, this is equivalent to increasing simultaneously the vacancy cost and the matching efficiency. As a result, in the new equilibrium, only $1/2$ of the number of old vacancies would be created. However, $\bar{x}$ would stay unchanged and so do the distribution of mismatch x, the wage, and output. Similarly, if we increase κ to 2κ and γ to 4γ, keeping ρ and δ unchanged, this is equivalent to increasing simultaneously the matching efficiency and the importance of a good match. As a result, $\bar{x}$ would shrink to $1/2$ of the old value but the distribution of $x/\bar{x}$ would remain unchanged. Any triple (κ, γ, K) can always be normalized to $(\kappa_0, \tilde{\gamma}, K_0)$ with κ_0 and K_0 pre-fixed. In the numerical simulations, we therefore set κ_0 and K_0 to 100 and 0.1 respectively.

Therefore, we only need to calibrate three parameters in the model: unemployment benefits B, on-the-job search efficiency ψ, and the joint value of $K\sqrt{\gamma}/\kappa$. The simplicity of the procedure allows us to see how robust our results are to different parameter values.

How large is ψ?

Our benchmark value of B is 0.4, which is the value used by Shimer (2005b). Our main result is however not sensitive to the value of B.

The remaining two unknown parameters are ψ and $K\sqrt{\gamma}/\kappa$. Our empirical targets are the job flow rates. Since the time unit is not explicitly defined, only the ratios of job flow rates are relevant. Let f_{ue} be the unemployment-to-employment flow, f_{eu} be the employment-to-unemployment flow, and f_{ee} be the employment-to-employment flow. Since unemployment is determined by the ratio between f_{ue} and f_{eu}, our calibration targets are the unemployment rate u and f_{ee}/f_{ue}. The target unemployment rates for the U.S. and the Netherlands are 5.6 and 7 percent, their long-run average.[11]

[11] Note that Gautier and Teulings (2015) take a different identification strategy. They allow for measurement error and estimate the unemployment rate to match observed wage dispersion and mismatch.

Nagypal (2005) and Gautier and Teulings (2015) derive how ψ can be estimated from observed labor market flows. since,

$$
\begin{aligned}
u f_{ue} &= 2\lambda\,(1-u)\,v\bar{x},\\
(1-u)\,f_{ee} &= 2\int_0^{\bar{x}} \psi\lambda\,(1-u)\,v g\,(x)\,x dx,\\
\frac{f_{ee}}{f_{ue}} &= \frac{u}{1-u}\left(\frac{u+\psi(1-u)}{\psi(1-u)}\ln\left(\psi\frac{1-u}{u}+1\right)-1\right).
\end{aligned}
$$

The first equation tells us that employment inflow equals the number of workers who met a vacancy in their matching set. The second equation relates the job-to-job flows to the employed workers who found a vacancy in their matching set and the final equation gives their ratio which no longer depends on λ. Gautier and Teulings (2015) argue that for the U.S., $\frac{f_{ee}}{f_{ue}}$ varies from 0.08 to 0.10 (depending on which data one uses and whether one targets the average or median worker) implying that $\psi \simeq 0.45$. Gautier et al. (1999), report for the Netherlands that $f_{ue} = 0.73$ and $f_{ee} = 0.061$, implying that the on-the-job search efficiency parameter, ψ, for the Netherlands is 0.35.

The calibration is summarized in Table 1.

When does decentralized wage setting perform better than CBA?

The effect of ψ

If we take the model seriously, it is an empirical question whether CBA is desirable and this mainly depends on ψ. Below, we show that for both the

Table 1: Benchmark values

		US	NL
B	leisure value	0.4	0.4
ψ	on-the-job search efficiency	0.40	0.35
$K\sqrt{\gamma}/\kappa$	parameter combination	0.0056	0.0077
u	unemployment rate	0.056	0.07
f_{ee}/f_{ue}	job flow rates ratio	0.08	0.0836

Netherlands and the US, ψ is above the threshold for which decentralized wage mechanisms perform better, implying that CBA reduces welfare.

First consider the US case. If we compare welfare for all three cases, Figure 1 shows that when $\psi < 0.25$, CBA outperforms decentralized wage setting. For higher values of ψ, CBA is not desirable. The reason is that it eliminates the incentive for on-the-job search which is crucial for social efficiency. When ψ is small, the benefits of on-the-job search are also small. Although firms will offer higher wages to the workers they prefer (workers anticipate this because they observe x),this benefit is small if they meet few workers. Therefore the benefits of CBA can in that case exceed the cost. From Figure 2, we can see that a similar conclusion holds for the Netherlands. For the empirical feasible parameters of $\psi \geq 0.3$, we must conclude that decentralized wage setting performs better than CBA in both countries.

As a robustness check, we use information from Gautier et al. (1999) for the Netherlands on different job and worker types to see if CBA could perform better in particular segments. Table 2 shows that in all segments $\psi > 0.24$ so that our conclusions that decentralized wage mechanisms are preferable holds for different segments.[12] Finally, job-offer-arrival rates for off- and on-the job search have been estimated in other papers (from which we can calculate ψ).

Table 2: On-the-job search parameter values for different categories

Category	p_{eu}	p_{ee}	ψ
simple job	4.08	6.67	0.5087
intermediate job	3.70	5.95	0.5237
complex job	4.41	4.92	0.2473
worker: low education	4.20	5.96	0.3835
worker: intermediate education	3.54	5.37	0.4798
worker: higher education	3.51	5.75	0.5591

Note: In calculating the relative on-the-job search efficiencies, we assume that the unemployment rate for each category is 8 percent.

[12] Considering different industries is harder because we can assign jobs to specific industries but not individual workers. Fallick and Fleischman (2004) do however report employment-to-employment (EE) worker flow rates. If we make the strong assumptions that each industry is a closed economy and that the unemployment rate is the same across industries, we can estimate ψ for different industries. For all 2 digit industries we find $\psi > 0.3$.

Unfortunately, their values vary substantially. Most (but not all) studies find values above 0.3. Dey and Flinn (2005) report values of around 0.17 for the US, Hornstein et al. (2011) find values around 0.35. Postel-Vinay and Robin (2002) estimate ψ for different occupations in France and find it to lie between 0.3 and 0.5. Lise et al. (2013) estimate ψ for different worker types and find it to equal 1.12 (less than high school), 0.52 (high school) and 0.25 (above high school). Belzil (1996) finds values close to one and Berg and Ridder (1998) find values greater than one.

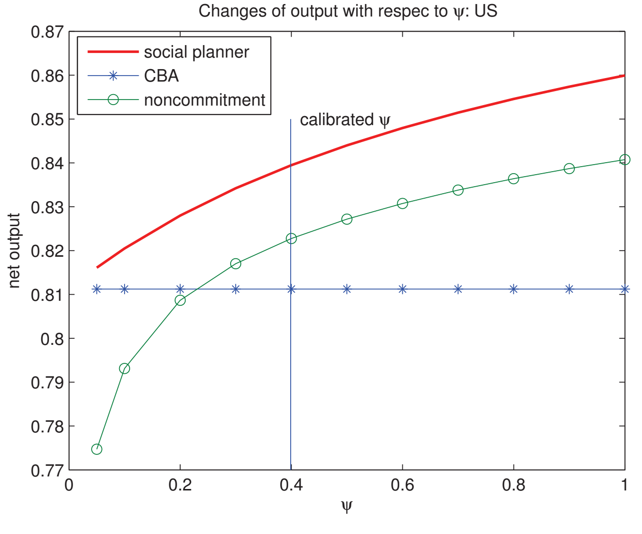

Figure 1: Robustness with respect to ψ: US

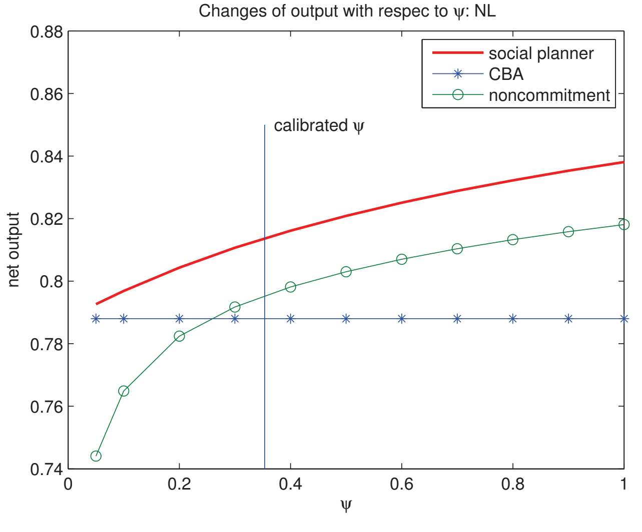

Figure 2: Robustness with respect to ψ: NL

The effect of γ

Another parameter of interest is γ, which governs the shape of the production function. Note from the above discussion that the effects of changes in $1/\kappa^2$ and K^2 are equivalent to changes in γ. If γ is 0, all jobs are identical, there will be no mismatch and the model reduces to Pissarides (2000). In this case, the best scenario of CBA coincides with the social planner's problem. From Figures 3 and 4, we can see that if we set ψ at its empirical value, for almost all positive values of γ, CBA performs worse than the decentralized case. Gautier and Teulings (2015) give evidence that $\gamma > 0$.[13]

The effect of β

In the above analysis, we have imposed that the bargaining power parameter is 0. Below we relax this and check the robustness of our results with respect to β. Since the empirical value of β is unknown, we present in Figure 5 and 6 four cases: $\beta = 0, 0.1, 0.4$ and 0.5. First consider the US case. Starting

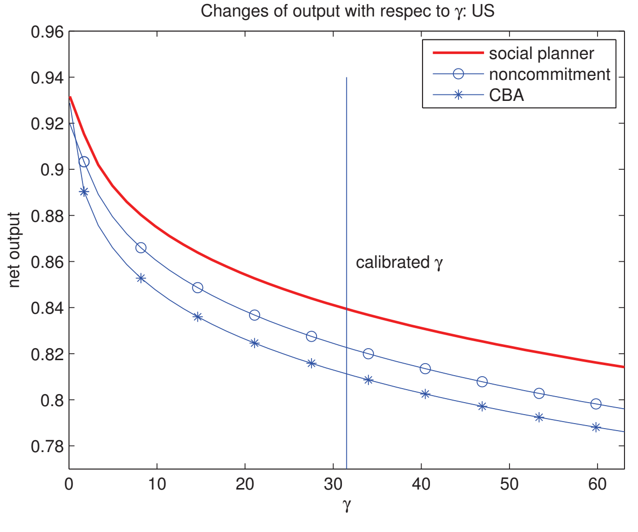

Figure 3: Robustness with respect to γ: US

[13]Note that γ is not independent on the scale of x so the values in our Figure cannot be directly compared to the ones in Gautier and Teulings (2015) which use a different normalization.

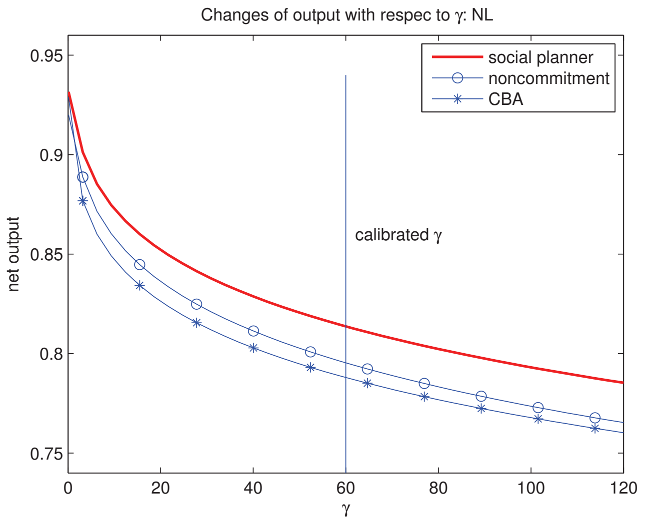

Figure 4: Robustness with respect to γ: NL

at $\beta = 0$, increasing β, reduces the business stealing externality. For $\beta = 0.1$, we see that the threshold of ψ above which decentralized bargaining is more efficient is around 0.1. At $\beta = 0.4$, the equilibrium outcome is very close to the social optimal and the business-stealing-externality is largely internalized. As we increase β further, worker's bargaining power becomes inefficiently high (for similar reasons as in Hosios (1990)) and we see that the decentralized equilibrium outcome performs worse. For $\beta > 0.75$, no vacancies are supplied and we enter the no-trade equilibrium. A too high bargaining power of workers leads to too little vacancy creation and too high unemployment. However, even for $\beta = 0.5$, the decentralized case performs much better than the CBA for most values of ψ (even if $\psi = 0.1$). Similar conclusions hold for the Netherlands.

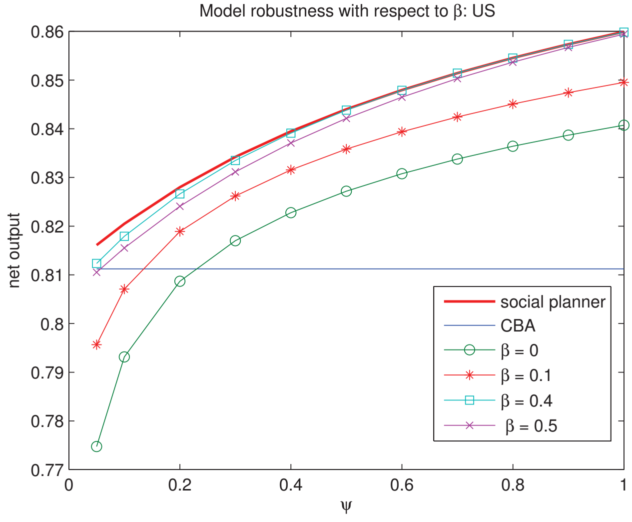

Figure 5: Robustness with respect to β: US

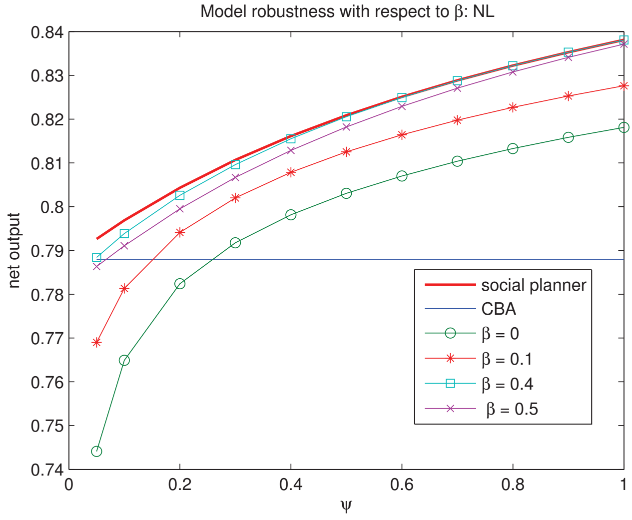

Figure 6: Robustness with respect to β: NL

2.5 Allowing for insider unions and pay scales

So far, we assumed that unions can only set one wage. In that case, decentralized wage setting schemes typically perform better than CBA even if the union's objective is to maximize social welfare. In this section we add some more realism to the model. This comes at the cost of greater complexity. First, in reality we observe that even under CBA there is wage dispersion within narrowly defined industries-occupation cells. Therefore, we allow unions to set pay scales. Second, we assume that the union maximizes the weighted average (over x) of the value of employment, $\mathbb{E}_G V^E(x)$. However, for a small discount rate, this turns out to be almost similar to maximizing the value of unemployment. On average, each worker spends the same amount of time in unemployment. With a positive discount rate, employed workers will put a bit more weight on the current state of employment. Therefore, the union has an incentive to not set wages too high because it realizes that its members may become unemployed in the future and care about the number of vacancies that is opened. Finally, we assume symmetric bargaining and set $\beta = 0.5$ but all our welfare conclusions continue to hold as long as $\beta \geq 0.2$. If $\beta \leq 0.1$, CBA with sufficiently many pay scales performs better than decentralized markets. Below, we formalize those extensions.

Suppose that the union can define pay scales consisting of N wages $\{w_1, \ldots, w_N\}$. After meeting a worker and learning the match productivity, $Y(x)$, the firm can select a wage from N. Given N, the union chooses the wages in N to maximize the expected value of employment. The decision of opening a vacancy is made by firms and after observing productivity x, the firm decides which wage from N to offer to its worker. This is in the spirit of the right-to-manage model and the efficient bargaining model.[14] As in the decentralized case, when x is small and so the match is productive, it is in the firm's interest to choose a relatively high wage from N in order to prevent the worker from quitting. So the piecewise constant wage curve is endogenous. Next we consider the union's problem in detail. For a given wage set, there are

[14] See Teulings and Hartog (1998) for a discussion of the various union wage-setting models.

$N-1$ thresholds, $\{x_1,\ldots,x_{N-1}\}$. At a given threshold x_k, the firm is indifferent between offering a wage w_{k+1} and w_k. I.e.,

$$\frac{y_k - w_k}{\rho+\delta+\psi\lambda v 2x_{k-1}} = \frac{y_k - w_{k+1}}{\rho+\delta+\psi\lambda v 2x_k},$$

Using $\kappa = 2\lambda/(\rho+\delta)$ gives

$$y_k - w_k = y_k - w_{k+1} - \frac{\psi\kappa v(x_k - x_{k-1})}{1+\psi\kappa v x_k}(y_k - w_{k+1}).$$

which can be written as,

$$\frac{w_{k+1}-w_k}{x_k - x_{k-1}} = -\frac{\psi\kappa v}{1+\psi\kappa v x_k}(y_k - w_{k+1}).$$

Note that if we let $N \to \infty$, this equation is the same as (2.13), the wage equation without commitment in the decentralized market with $\beta = 0$. The difference between the union setting and the decentralized case is that the union can set the lowest wage in the market while in the decentralized setting, the lowest wage is determined by the reservation wage condition, i.e. the wage where the worker is indifferent between being employed and unemployed. The union will set the lowest wage in N at a level that makes the value of employment at this wage exceed the value of being unemployed. The model is again solved numerically for different values of ψ and also for different pay scales. Figure 7 shows that now also for CBA, a higher ψ increases social welfare because firms that need a particular worker type can now offer this worker type a higher wage and this encourages on-the-job search.

A number of interesting conclusions can be drawn from Figure 7. First, it hardly matters for social welfare whether the union maximizes V^E or V^U. Since employed workers can loose their job, unions that maximize V^E, also take into account that high wages reduce job creation. Second, the pay scales should contain sufficiently many wages. Here it turns out that having four wages generates more welfare than two wages and almost the same as infinitely many wages. So we do not need a pay scale scheme with more than four wages. A remaining question is why can't the union reach the social optimum when it uses four or more wages in its pay scale even if it maximizes V^U? As long as the wage curve $w(x)$ is downward sloping, only $\bar{x}$ and v matter for social efficiency. The planner could simultaneously pick $(\bar{x}, v)$ while the union can

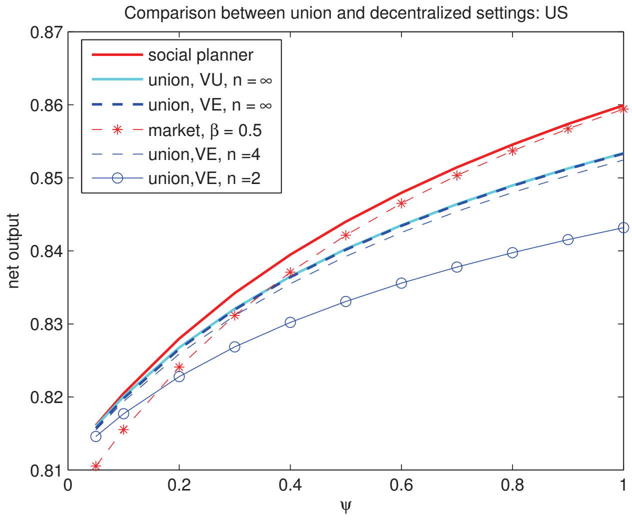

Figure 7: Net outputs for different number of wages

only set the lowest wage which will affect both $\bar{x}$ and v. The second question is why the decentralized case with $\beta = 0.5$ still performs better than the CBA case for sufficiently large ψ. To understand this, consider two extreme cases: $\psi = 0$ and $\psi = 1$. With $\psi = 0$, the union can only set one wage, and we are back in the case considered before. In this case, the union can reach the social planner's solution (it can satisfy the Hosios condition), so CBA performs better than the decentralized case. For small values of ψ, a similar logic applies. For $\psi = 1$, the union will take the business-stealing externality into account because it does not want that too many resources in the economy are spent on vacancy creation. If the union sets the lowest wage at B, worker's behave as they should (accept all wages above B and move if they find a better match) but because of the business stealing externality, too many vacancies will then be created. So the union must sacrifice some efficiency on the acceptance margin to reduce the excess resources that are spent on vacancy creation. If workers have no bargaining power ($\beta = 0$), CBA performs better than the decentralized market but in the more realistic case that workers have positive bargaining power (i.e. $\beta = 0.5$), the wage curve in the market will be steeper than the one under

CBA because the union can only define pay scales while a firm can decide in which pay scale it places the worker. The steeper wage curve that arises in the decentralized case with $\beta = 0.5$, mitigates the business stealing externality. In fact, we find that for $0.1 \le \beta \le 0.7$, decentralized wage setting outperforms CBA, irrespective of the number of pay scales, N.

2.6 Final remarks

In this chapter we used a search model with ex ante heterogeneity to derive a threshold on the relative efficiency of on-the-job search below which centralized bargaining agreements are desirable. We first considered the extreme case where CBA implied a fixed wage within an industry. We relaxed this assumption by allowing the union to set pay scales. The empirical estimates for both the Netherlands and the US in the relevant segments are above the threshold (below which CBA is desirable), implying that the cost of CBA (the economy moves slower to the optimal allocation because wages lose their signaling function) exceed the potential benefits (the business-stealing externality can be internalized). Our results turn out to be robust for changes in bargaining power and the parameter that measures the importance of a precise match.

In our model we treated search intensity as an exogenous parameter. Endogenizing search intensity goes beyond the scope of this chapter but it would likely make the decentralized case more desirable. Workers who are badly matched would search more intensively for more productive jobs (if firms would offer those workers a higher wage).

Finally, we find that a union that maximizes the expected value of employment behaves very similarly to a union that maximizes the value of unemployment because (infinitely lived) employed workers will at some point in time also become unemployed. In many European countries, unions are populated by a large fraction of well protected old workers, who typically put less weight on job creation because they are unlikely to become unemployed. Again, allowing for this would complicate our model but since it would make the decentralized wage setting case even more desirable it would not change our main conclusions.

Appendix

Derivation of the asset values

Totally differentiating (2.7) yields

$$V_x^E(x) = \frac{W_x(x)}{\rho + \delta + 2\psi\lambda x}. \tag{.21}$$

Implying that

$$V^E(x) = \int_0^x \frac{W_x(z)}{\rho + \delta + 2\psi\lambda z} dz + C_0.$$

Integrating by parts yields

$$V^E(x) = \frac{W(x)}{\rho + \delta + 2\psi\lambda x} - \frac{W(0)}{\rho + \delta} + 2\lambda\psi \int_0^x \frac{W(z)}{\left(\rho + \delta + 2\psi\lambda z\right)^2} dz + C_0. \tag{.22}$$

Evaluating (.22) at $x = 0$ gives an initial condition that can be used to solve for C_0:

$$C_0 = V^E(0) = \frac{W(0)}{\rho + \delta} + \frac{\delta}{\rho + \delta} V^U.$$

Substituting this back into (.22) yields the desired expression. Let $\mathrm{E}_x W \equiv \int_0^{\bar{x}} g(x) W(x) dx$ be the expected wage of a filled job. Evaluate (2.7) at $\bar{x}$ and use the definition of g in (2.5) to get

$$\rho V^E(\bar{x}) = \rho V^U = \frac{W(\bar{x}) + \psi\bar{x}\mathrm{E}_x W}{1 + \psi\bar{x}} = \frac{uW(\bar{x}) + \psi(1-u)\mathrm{E}_x W}{u + \psi(1-u)}. \tag{.23}$$

Next, note that the right-hand sides of (2.6) and (2.9) are equal, which can be used to get an expression for $\int_0^z \left[V^E(x) - V^U\right] dx$. Substitution of this expression into (2.8) gives ρV^U as a function of $W(\bar{x})$, which can be eliminated by solving (2.9) for $W(\bar{x})$. This gives,

$$\rho V^U = \frac{B + \bar{x}\mathrm{E}_x W}{1 + \bar{x}} = uB + (1-u)\mathrm{E}_x W, \tag{.24}$$

where the final step uses (2.2).

The free-entry condition implies that the option value of a vacancy of type c must be equal to K. Hence, by defining $\mathrm{E}_x Y \equiv \int_0^{\overline{x}} g(x)\, Y(x)\, dx$, we obtain

$$\begin{aligned} K &= 2\lambda \int_0^{\overline{x}} \{u + \psi(1-u)[1 - G(x)]\} \frac{Y(x) - W(x)}{\rho + \delta + 2\psi\lambda x} dx \\ &= (1-u)(\mathrm{E}_x Y - \mathrm{E}_x W). \end{aligned}$$

The first term in the integrand is the effective labor supply, $u + \psi(1-u)[1 - G(x)]$ for a vacancy of type x. It is equal to the number of unemployed, u, plus the number of workers employed in jobs with a mismatch indicator that exceeds x, $(1-u)[1 - G(x)]$. The second factor is the discounted value of a filled vacancy. Just as in the wage equation, we discount current revenue $Y(x) - W(x)$ by the discount rate ρ plus the separation rate δ plus the quit rate $2\psi\lambda x$. The second line follows from substituting (2.2) and (2.5) in the top equation. In free entry equilibrium, all firm profits are spent on vacancy creation.

Wages under no commitment for general β

In this appendix we derive wages for general bargaining power and no commitment. The derivations for commitment are very similar. Let $\widehat{F}(W)$ be the survival function of the wage-offer distribution (which for well known reasons cannot have mass points, see Burdett and Mortensen, 1998). $W(x)$ maximizes the following product,

$$W(x) = \arg\max_W \left[\hat{V}^E(W) - V^U\right]^\beta \left[\frac{Y(x) - W}{\rho + \delta + 2\psi\lambda v \bar{x} \widehat{F}(W)}\right]^{1-\beta}.$$

Since $\hat{V}^E_W(W) = \left[\rho + \delta + 2\psi\lambda v \overline{x} \widehat{F}(W)\right]^{-1}$, we can write the first-order condition as,

$$\begin{aligned} \beta[Y(x) - W(x)] = (1-\beta)\left[\hat{V}^E[W(x)] - V^U\right] \times \\ \left[\rho + \delta + 2\psi\lambda v \bar{x} \widehat{F}[W(x)] + 2\psi\lambda v \bar{x} \widehat{F}_W[W(x)][Y(x) - W(x)]\right], \end{aligned}$$

Since $Y(x)$ is decreasing in the distance x, $W(x)$ is decreasing in x. Therefore,

$$\begin{aligned} V^E(x) &\equiv \hat{V}^E[W(x)], \\ 2vx &= \widehat{v}[W(x)]. \end{aligned}$$

The second line follows from the uniformity of unemployed workers and of vacancies. Using the chain rule gives,

$$\begin{aligned} V_x^E(x) &= \hat{V}_W^E[W(x)]\, W_x(x), \\ 2v &= \widehat{v}_W[W(x)]\, W_x(x). \end{aligned} \tag{.25}$$

Substitution of (.25) and the expression for $Y(x)$ in the first order condition and rearranging terms yields

$$W_x(x) = \frac{(1-\beta)\psi\kappa v(Y(x)-W(x))R(x)}{\beta(Y(x)-W(x))-(1-\beta)(1+\psi\kappa v x)R(x)}, \tag{.26}$$

where $R(x) = (\rho+\delta)(V^E(x) - V^U)$.

Gautier et al. (2010) show that,

$$V^E(x) = \frac{W(x)}{\rho+\delta+2\psi\lambda v x} + \frac{\delta}{\rho+\delta}V^U + 2\psi\lambda v\int_0^x \frac{W(z)}{(\rho+\delta+2\psi\lambda v z)^2}dz$$

Differentiating both sides with respect to x, gives

$$\frac{d}{dx}V^E(x) = \frac{W'(x)}{\rho+\delta+2\psi\lambda v x}$$

This implies that,

$$\frac{d}{dx}R(x) = \frac{W'(x)}{1+\psi\kappa v x}, \tag{.27}$$

together with the initial condition $R(\bar{x}) = 0$. Equations (.26) and (.27) together imply that for general β, the individual ρ, δ, or λ doesn't matter. Only κ enters the equations.

Using Equations (.26) and (.27), we solve the differential equation numerically.[15]

[15] The program can be received upon request.

CHAPTER 3

Meeting Technologies, Heterogeneity, and Competing Mechanisms[16]

3.1 Introduction

One of the fundamental questions in the economic literature concerns the choice of a trading mechanism by a seller who wishes to sell a good. A large literature in mechanism design analyzes this question in the context of a monopolistic seller and often finds that auctions dominate posted prices. Recent work by Eeckhout and Kircher (2010b) however points out that in a market setting in which sellers compete for buyers with private valuations by posting mechanisms, the process that governs meetings between buyers and sellers—i.e., the *meeting technology*—is crucially important for the choice of mechanism. If buyers randomly select a seller and sellers are unconstrained in the number of buyers that they can meet (urn-ball meetings), then auctions are indeed useful instruments to identify the buyer with the highest valuation. The constrained efficient equilibrium in this case consists of a single market in which all sellers post auctions and all buyer types pool, as this maximally spreads high-type buyers across sellers. However, if sellers are, for example,

[16]This chapter is based on Cai et al. (2015)

time-constrained and can only meet one buyer at a time (bilateral meetings), low-type buyers may crowd out high-type buyers. In that case, sellers prefer to post prices, which induces perfect separation of buyers into homogeneous submarkets. In this chapter we study how the meeting technology affects posted mechanisms. Specifically, we analyze an environment based on Eeckhout and Kircher (2010b), in which a continuum of buyers and sellers try to trade subject to the frictions generated by an arbitrary meeting technology. Unlike Eeckhout and Kircher (2010b), who restrict attention to two types of buyers, we allow for arbitrary distributions of valuations. We derive necessary and sufficient conditions on the meeting technology under which it is optimal to have (i) a separate market for each type of buyer, or (ii) a single market with all buyer types (pooling), for any buyer type distribution.[17]

Although the assumption of either bilateral or urn-ball meetings is nearly universal in the search literature[18], neither technology is necessarily an adequate description of real-life markets. In many cases, it might be more realistic to consider a technology which allows a seller to meet and learn the type of multiple but not all buyers who are interested in matching with him. For example, in the labor market, screening costs prevent firms from learning the type of all their applicants.[19] In the housing market, viewings are often costly for the sellers and this may affect the meeting rate (sellers may want to meet with only a subset of the buyers or only organize a viewing if there are at least i and at most j buyers).[20] In the marriage market, meetings typically used to be bilateral but since the introduction of on line dating, many-to-one meetings are now also common. In the market for goods and services, procurers can typically meet multiple contractors per period while car dealers typically meet with one buyer at a time.[21] For products sold online, meetings are sometimes

[17]Eeckhout and Kircher (2010b) move beyond bilateral and urn-ball by considering general meeting technologies, but stop short of characterizing necessary and sufficient conditions for pooling or separating equilibria. We discuss the connection with their work in detail in section 3.5. The only other paper that studies general meeting technologies, Lester et al. (2015), assumes homogeneous buyers and is therefore silent on this topic.

[18]Bilateral meetings can be found in e.g. Moen (1997), Guerrieri et al. (2010), and Menzio and Shi (2011). Urn-ball is used in e.g. Peters (1997), Burdett et al. (2001), Shimer (2005a), and Albrecht et al. (2014).

[19]See Fraja and SÃąkovics (2001), Lester and Wolthoff (2014), and Wolthoff (2015).

[20]Albrecht et al. (2014) and Lester and Wolthoff (2014) allow for many-to-one meetings in the housing market.

[21]See for example Kim and Kircher (2012).

many to one (eBay) and sometimes bilateral (Amazon). Finally, schools often require a minimum number of applicants before a new class is opened or some boat companies require a minimum number of customers before they operate, so buyers can impose positive spillovers on each other.[22] All those different environments give rise to different meeting technologies which in turn affect the sellers choices of mechanisms.

Our main methodological innovation is that in order to analyze these general meeting technologies, we introduce a new function, ϕ. This new function is an alternative representation of the meeting technology and it specifies the probability for a seller to meet at least one buyer from a given subset. This function depends on the buyer-seller ratio and the fraction of buyers in this subset. We show that the expected surplus at a seller only depends on the integral of this function over buyer valuations, x (where the relevant subset equals the expected number of buyers with a valuation above x). In a seminal paper, Myerson (1981) formulated and solved the "optimal auction design problem" among all possible ways of selling the good, by introducing the virtual valuation function. His tools have been widely applied in auction theory and other areas where private information plays a key role. We use ϕ to solve the optimal auction design problem in a competitive environment for any meeting technology. Although this chapter provides only a further step toward understanding the role of the meeting technology on the choice of mechanism and the effects on market segmentation, we expect that our methodology will lead to more progresses along this line. It turns out that there is a simple relation between ϕ and the virtual valuation. In standard auction theory, the seller's payoff is obtained by integrating a buyer's virtual valuation (which equals the expected value of the second highest valuation) against the distribution of highest valuations. We show here that for a given buyer-seller ratio, λ, the latter term can simply be replaced by $1-\phi(\lambda(1-F(x)),\lambda)$, which simply equals the probability that no buyers arrive with valuations above x. So the introduction of ϕ adds a lot of generality to the competing mechanism literature at relatively low cost.

After describing this environment in detail in section 3.2, we start our analysis in section 3.3 by considering the trade-off of a social planner between

[22] See for example Geromichalos (2012)

the desire to spread high-type buyers as much as possible and the risk of them being crowded out by low-type buyers.

Our first result concerns the optimality of perfect separation. We find that this outcome is not very robust: bilateral meetings are not only sufficient, but also necessary. That is, if one moves away from bilateral meetings by allowing a seller to meet multiple buyers (potentially with arbitrary small probability), then there exist distributions of buyer valuations for which perfect separation is no longer efficient. Intuitively, full separation does not exploit the efficiency gains that arise from sellers ranking multiple buyers: with homogeneous submarkets, any meetings beyond the first are irrelevant since they always present the seller with a clone of the buyer that he has already met.

Although the necessity of bilateral meetings is a new result in the literature, it is perhaps not very surprising. Most of our attention therefore goes out to the optimality of a single market in which all agents participate. We show that this outcome is more robust. To be precise, a single market is the efficient outcome if and only if ϕ is concave and in this case we call the meeting technology exhibits "love for variety." This condition guarantees that social surplus can be increased by merging any two submarkets, irrespective of their composition. Intuitively, given that ϕ enters linearly in the seller's surplus, concavity of ϕ (or love for variety) is a necessary and sufficient condition to pool all buyer types in one segment. Love for variety is satisfied by the urn-ball meeting technology, which explains why pooling is the efficient outcome in e.g. Peters and Severinov (1997), but we also describe a number of other meeting technologies that exhibit this property.

In the second half of section 3.3, we discuss how both the pooling and the separating outcome can be decentralized by each seller posting a second-price auction, combined with a meeting fee to be paid by (or to) each buyer meeting him.[23] Intuitively, in a large market, sellers take buyers' equilibrium payoffs as given, making sellers the residual claimant on any extra surplus that they create and providing them with an incentive to post efficient mechanisms. Auctions guarantee that the good is allocated efficiently, while the meeting fees price any positive or negative externalities in the meeting process, providing all

[23]As we explain in more detail below, this mechanism reduces to posted prices when meetings are bilateral.

agents with a payoff equal to their social contribution. Although our baseline model assumes an exogenous number of sellers, these results imply that the conclusion of Albrecht et al. (2014) that seller entry is efficient under the urn-ball meeting technology can be extended to our environment.[24] In section 3.4 we show how a seller's surplus depends on both the virtual valuation and on ϕ.

We conclude our analysis by comparing our findings to existing results in section 3.5. In particular, we express different classes of meeting technologies like: (i) invariance as introduced by Lester et al. (2015), and (ii) non-rivalry as introduced by Eeckhout and Kircher (2010b), in terms of our new function ϕ and relate them to each other. We show that invariance is a sufficient (but not a necessary) condition for love for variety, while non-rivalry is a necessary (but not a sufficient) condition, and we explain why this is the case. Finally, the appendix contains all proofs.

3.2 Environment

Agents and Preferences. Consider a static economy populated by a measure 1 of sellers, indexed by $j \in [0,1]$, and a measure $\Lambda > 0$ of buyers. Both types of agents are risk-neutral. Each seller possesses a single unit of an indivisible good, for which each buyer has unit demand.[25] All sellers have the same valuation for their good, which we normalize to zero. A buyer's valuation is an independent draw from a distribution $F(x)$ with $0 \le x \le 1$.[26] We impose no additional structure on $F(x)$, although we will sometimes pretend that all buyers have either a low valuation x_1 or a high valuation x_2 when describing the intuition behind our results.[27] Buyers observe their valuation before making any decisions. An agent's payoff is the sum of (i) his monetary transfers and (ii) his valuation if he possesses the good at the end of the period (and zero otherwise).

[24] We formally establish this in the online appendix, which contains a number of additional results.

[25] Although we analyze a goods market, it is straightforward to cast our model in a labor market setting in which homogeneous firms post menus of wages to attract workers who differ in their productivity, as in Shi (2006). All our results carry over to such an environment.

[26] The assumption that all buyers have a (weakly) higher valuation than the seller is standard as well as innocuous. Buyers with lower valuations would simply never trade.

[27] It is worth highlighting that none of our results are driven by the requirement that they should hold for all $F(x)$. That is, our results remain the same if we consider the *weaker* requirement that they should hold for all $F(x)$ with only two points of support.

Search. In order to attract buyers, each seller posts and commits to a direct mechanism. As in Lester et al. (2015), a mechanism specifies an extensive form game that determines for each buyer i a probability of trade and an expected payment as a function of: (i) the total number n of buyers that meet with the seller; (ii) the valuation x_i that buyer i reports; and (iii) the valuations x_{-i} reported by the $n-1$ other buyers.[28]

All identical mechanisms are treated symmetrically by buyers and are therefore said to form a *submarket*. After observing all submarkets, each buyer chooses the one in which he wishes to attempt to match.[29] As standard in the literature (see e.g. Shimer, 2005a), we capture the anonymity of the large market by assuming that i) sellers can condition their strategies on the actions of buyers but not on their identities, and ii) identical buyers must use identical mixed strategies in equilibrium.

Meeting Technology. Conditional upon the choice of a submarket, meetings between buyers and sellers are governed by a frictional process, the *meeting technology*.[30] To introduce this process, suppose a submarket is visited by a measure b of buyers and a measure s of sellers. Defining $\lambda = b/s$ as the *queue length* (or the inverse of the market tightness) in this submarket, we then follow Eeckhout and Kircher (2010b) and Lester et al. (2015) by specifying that a seller meets $n \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$ buyers with probability $P_n(\lambda)$.

Assumptions. We impose a number of restrictions on the meeting technology. First, it must satisfy $\sum_{n=0}^{\infty} nP_n(\lambda) \leq \lambda$, because the number of meetings cannot exceed the number of buyers in the submarket. Second, we assume that $P_n(\lambda)$ is twice-continuously differentiable. Finally, we maintain the assumption of Eeckhout and Kircher (2010b) that, within each submarket,

[28] In line with most of the literature, we abstract from mechanisms that condition on other mechanisms present in the market. See Epstein and Peters (1999) and Peters (2001) for a detailed discussion.

[29] The assumption that a buyer can meet only one seller per period is standard in the directed search literature. See Albrecht et al. (2006), Galenianos and Kircher (2009), Kircher (2009), Gautier and Holzner (2014) and Wolthoff (2015) for papers that relax this assumption.

[30] Note that, as in Eeckhout and Kircher (2010b) and Lester et al. (2015), there exists a distinction between meeting with a seller (i.e. participating in his mechanism) and matching (i.e. acquiring his good).

the meeting technology allocates buyers to sellers in a way that is independent of types.[31] In other words, if a measure $\mu \in [0, \lambda]$ of the buyers in the submarket are labeled "blue," then the probability for a seller to meet i blue buyers and $n - i$ other buyers equals

$$P_n(\lambda) \binom{n}{i} \left(\frac{\mu}{\lambda}\right)^i \left(1 - \frac{\mu}{\lambda}\right)^{n-i}.$$

Alternative Representation. It will prove useful to define a function $\phi(\mu, \lambda)$, which represents the probability that a seller with a queue μ of blue buyers and a queue $\lambda - \mu$ of other buyers meets at least one blue buyer. Hence, given the above assumption regarding type-independent allocation of buyers, $\phi(\mu, \lambda)$ equals

$$\phi(\mu, \lambda) = 1 - \sum_{n=0}^{\infty} P_n(\lambda) \left(1 - \frac{\mu}{\lambda}\right)^n. \tag{3.1}$$

The function $\phi(\mu, \lambda)$ provides an alternative representation of $P_n(\lambda)$. To see this, let $m(z, \lambda) \equiv \sum_{n=0}^{\infty} P_n(\lambda) z^n = 1 - \phi(\lambda(1-z), \lambda)$ denote the probability-generating function of $P_n(\lambda)$. It then follows from the properties of probability-generating functions that $P_n(\lambda) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} m(z, \lambda)\Big|_{z=0} = \frac{(-\lambda)^n}{n!} \frac{\partial^n}{\partial \mu^n} \left(1 - \phi(\mu, \lambda)\right)\Big|_{\mu=\lambda}$. To simplify notation, we will often omit the arguments of ϕ and use subscripts to indicate its partial derivatives.

Examples of Meeting Technologies. For future reference, it will be useful to formally define a few examples of meeting technologies that satisfy all our assumptions.

1. *Urn-Ball.* First explored by Butters (1977) and Hall (1977), this technology specifies that—within a submarket—each buyer is randomly allocated to one of the sellers. As a result, the number of buyers that meet a particular seller follows a Poisson distribution with mean equal to the queue length λ. That is, $P_n(\lambda) = e^{-\lambda} \frac{\lambda^n}{n!}$, which yields $\phi(\mu, \lambda) = 1 - e^{-\mu}$.[32]

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[31] Of course, the equilibrium (or planner's) allocation of buyers to *submarkets* can depend on types.

[32] To keep the exposition concise, we omit the (straightforward) derivation of $\phi(\mu, \lambda)$ for each example.

2. *Bilateral.* With this technology, each seller meets at most one buyer, i.e. $P_0(\lambda) + P_1(\lambda) = 1$ or $\phi\left(\mu, \lambda\right) = P_1(\lambda)\frac{\mu}{\lambda}$. A potential micro-foundation consists of randomly pairing agents and keeping only pairs that consist of one buyer and one seller, yielding $P_1(\lambda) = \frac{\lambda}{1+\lambda}$.[33]
3. *Pairwise Urn-Ball.* This technology, described by Lester et al. (2015), is a variation on the urn-ball technology. Buyers first form pairs, after which each pair is randomly assigned to a seller in the submarket. That is, $P_n(\lambda) = 0$ for $n \in \{1, 3, 5, \ldots\}$ and $P_n(\lambda) = e^{-\lambda/2}\frac{(\lambda/2)^{n/2}}{(n/2)!}$ for $n \in \{0, 2, 4, \ldots\}$, which implies $\phi(\mu, \lambda) = 1 - e^{-\mu\left(1-\frac{1}{2}\frac{\mu}{\lambda}\right)}$.
4. *Multi-Platform.* This technology consists of two platforms or rounds. In the first round, all b buyers and a fraction $0 < \alpha < 1$ of the s sellers in a submarket attempt to meet according to the bilateral technology described above. The $\frac{b}{b+\alpha s}b = \frac{\lambda}{\lambda+\alpha}b$ buyers who fail to meet a seller then participate in the second round, in which they meet the remaining $(1-\alpha)s$ sellers according to an urn-ball process. That is,

$$P_n(\lambda) = \begin{cases} \alpha\frac{\alpha}{\lambda+\alpha} + (1-\alpha)e^{-\xi} & \text{if } n = 0 \\ \alpha\frac{\lambda}{\lambda+\alpha} + (1-\alpha)\xi e^{-\xi} & \text{if } n = 1 \\ (1-\alpha)\frac{\xi^n e^{-\xi}}{n!} & \text{if } n \in \{2, 3, \ldots\}, \end{cases}$$

where $\xi = \frac{\lambda^2}{(1-\alpha)(\lambda+\alpha)}$ is the queue length in the second round. This yields $\phi\left(\mu, \lambda\right) = \alpha\frac{\mu}{\lambda+\alpha} + (1-\alpha)\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right)$.[34]

[33]This micro-foundation can be found in the money search literature (see e.g. Kiyotaki and Wright, 1993). Some papers in the labor search literature provide an alternative, consisting of an urn-ball process augmented with the constraint that each seller can only contact one random buyer among the ones that wish to meet him, such that $P_1(\lambda) = 1 - e^{-\lambda}$ (see e.g. Albrecht et al., 2006; Galenianos and Kircher, 2009; Gautier and Wolthoff, 2009; Gautier et al., 2015).

[34]While this technology may seem more involved than the other examples, the two-round structure actually resembles the meeting process in various real-life markets: (i) buyers who cannot find a product at the local bazaar may subsequently submit a bid at an online auction site; (ii) workers who have trouble finding a job are often put in touch with firms by a public employment agency; and (iii) singles who fail to meet someone in a bar may subscribe to a dating website. Admittedly, the analogy is not perfect because terms of trade may differ across platforms in reality, which is ruled out here by the definition of a submarket.

3.3 Planner's Problem and Market Equilibrium

In this section, we first analyze the problem of a social planner whose objective is to maximize social surplus, while being constrained by the coordination frictions of the environment. Subsequently, we discuss how the planner's solution can be decentralized.

Planner's Problem

The planner's problem consists of two parts. First, the planner has to allocate buyers and sellers to submarkets, creating a queue length and a distribution of buyer types at each seller. Second, the planner has to specify how trade will take place after buyers arrive at sellers. We solve these stages in reverse order.

Trading Rule. Once a number of buyers $n \in \mathbb{N}_1$ has arrived at a seller, surplus is clearly maximized by allocating the good to the buyer with the highest valuation. The following lemma establishes the expected surplus generated by this trading rule.

Lemma 3. *The surplus created by a seller with a queue λ of buyers whose types are distributed according to the distribution $G(x)$ equals*

$$S(\lambda, G) = \int_0^1 \phi(\lambda(1 - G(x)), \lambda)\, dx.$$

Allocation of Buyers. Now consider the allocation of buyers to sellers. For each seller $j \in [0,1]$, the planner chooses—with a slight abuse of notation—a queue length $\lambda(j)$ and a distribution of buyer types $G(j,x)$ to maximize total surplus

$$\mathscr{S} = \int_0^1 S(\lambda(j), G(j,x))\, dj.$$

Of course, the planner cannot allocate more buyers of a certain type than are available. Formally, $\int_0^1 \lambda(j)\nu(j,B)\, dj \le \Lambda \nu_F(B)$ for any Borel-measurable set B, where ν_F is the measure associated with F and $\nu(j,\cdot)$ is the measure associated with $G(j,\cdot)$.

In general, the planner's solution and the market equilibrium can be rather complex, consisting of potentially many submarkets, each with a subset of all buyer types, with the outcome depending on the exact distribution of valuations. We therefore derive necessary and sufficient conditions for the two tractable cases typically considered in the literature, perfect separation and perfect pooling of types for all type distributions.

Separation. We first establish that bilateral meetings are a necessary and sufficient condition for full separation of buyer types.

Proposition 4. *Bilateral meetings are a necessary and sufficient condition for the planner to create a separate submarket for each type of buyer under any type distribution* $F(x)$.

Sufficiency of bilateral meetings for full separation is a well-known result in the literature: a separate submarket for each active buyer type avoids the high degree of crowding-out that arises if high-type and low-type buyers visit the same submarket and sellers meet one of both at random.[35] Necessity is however—to the best of our knowledge—a new result.[36] To understand the intuition, suppose that a seller can meet two or more buyers with positive probability. With full separation, any meetings beyond the first are irrelevant—as a seller will always meet a clone of the first buyer—and the gain in surplus relative to a bilateral technology is zero. Letting one high-type and one low-type buyer swap submarket, however, provides a way to increase surplus. After all, there is a positive probability that both these buyers meet sellers who meet other buyers as well. In that case, the buyers' joint contribution to surplus was 0 before the swap, but $x_2 - x_1$ after the swap (generated by the high-type buyer; the low-type buyer still contributes 0).[37]

[35] A planner may of course keep the lowest types out of the market altogether if a marginal seller can generate more surplus in a different submarket (see e.g. Eeckhout and Kircher, 2010b).

[36] Note that this result does not contradict Eeckhout and Kircher (2010b) who discuss how full separation dominates full pooling even when meetings are not strictly bilateral. The difference arises because they compare full separation against full pooling for a given (two-type) distribution, while we compare it against arbitrary other allocations for arbitrary distributions, making the set of technologies for which full separation is optimal necessarily smaller.

[37] Of course, this argument is complete only if a buyer's probability to meet a seller is the same in both submarkets, or otherwise the change in surplus associated with changing the buyers' meeting probabilities has to be taken into account. The proof of the proposition therefore focuses on the case in which both types of buyers have valuations that are arbitrarily close.

Pooling. To state our main result regarding pooling, we define a novel property of meeting technologies, which we call "love for variety."

Definition 5. *A meeting technology exhibits* love for variety *if and only if* $\phi(\mu,\lambda)$ *is concave in* (μ,λ)*, i.e.*

$$\phi_{\mu\mu}\phi_{\lambda\lambda} \geq \phi^2_{\mu\lambda}, \tag{3.2}$$

for all $0 \leq \mu \leq \lambda < \infty$.[38]

The following proposition then establishes that love for variety is closely related to the optimality of a single market.

Proposition 6. *Love for variety is a necessary and sufficient condition for the planner to send all agents to the same market under any type distribution* $F(x)$.

The intuition for this result is straightforward.[39] Since the surplus created by a submarket is linear in ϕ, love for variety (i.e. concavity of ϕ) implies that merging any two submarkets always leads to a higher surplus. The condition is not only sufficient but also necessary, because the result should hold for arbitrary type distributions. We provide a detailed discussion of love for variety in section 3.5, after considering decentralization first.

Market Equilibrium

We now show that the solutions to the planner's problem characterized in the previous subsection can be decentralized as a directed search equilibrium in which each seller posts a second-price auction combined with a meeting fee τ to be paid by each buyer meeting him.[40] As the argument is standard, we keep the exposition brief.

To define equilibrium, let $R(\tau,\lambda,G)$ denote the expected payoff of a seller who posts a second-price auction with a meeting fee τ and attracts a queue of buyers (λ,G). Further, let $U(x,\tau,\lambda,G)$ denote the expected payoff of a buyer with valuation x who visits this seller. Each seller aims to maximize his revenue

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[38]Condition (3.2) is necessary and sufficient for concavity, since $\phi_{\mu\mu} < 0$ for all non-bilateral technologies.

[39]Here the advantage of using ϕ becomes apparent; the equivalent condition in terms of P_n, which we derive in the online appendix, is far less simple and intuitive.

[40]The meeting fee can be negative, turning it into a meeting subsidy paid *to* each buyer.

R, but must take into account that his queue (λ, G) is endogenously determined and depends on the fee τ that he posts. To formalize this, let $\overline{U}(x)$ denote the *market utility* of a buyer with valuation x, i.e. the highest expected payoff that this buyer can obtain in equilibrium. Optimal search behavior then implies that a seller is visited by buyers of type x with positive probability only if he offers them their market utility. Hence, we can define an equilibrium as follows.

Definition 7. *A directed search equilibrium is a second-price auction, a meeting fee $\tau(j)$ and a queue $(\lambda(j), G(j,x))$ for each seller $j \in [0,1]$, combined with a market utility $\overline{U}(x)$ for each type of buyer x, such that ...*

1. *each tuple $(\tau(j), \lambda(j), G(j,x))$ maximizes $R(\tau, \lambda, G)$ subject to $U(x, \tau, \lambda, G) \leq \overline{U}(x)$, with equality for each x in the support of G;*
2. *aggregating queues across sellers does not exceed the total measure of buyers of each type.*

Note that a second-price auction does not generate any revenue if meetings are bilateral. Hence, the meeting fees essentially act as posted prices in that case. That posted prices suffice to decentralize the planner's solution in such an environment follows readily from Eeckhout and Kircher (2010a,b).[41] In the following proposition, we therefore focus on the decentralization of the planner's solution for technologies that satisfy love for variety.

Proposition 8. *When the meeting technology exhibits love for variety, all sellers posting a second-price auction and a meeting fee equal to*

$$\tau(j) = -\frac{\int_0^1 \phi_\lambda(\Lambda(1-F(x)), \Lambda)\, dx}{\phi_\mu(0, \Lambda)},$$

attracting a queue $(\lambda(j), G(j,x)) = (\Lambda, F(x))$, is a directed search equilibrium. This equilibrium decentralizes the social planner's problem.

The intuition for this result is partly the same as in many other directed search models. Since sellers take buyers' equilibrium payoffs as given, they are the residual claimant on any surplus that they create. This provides them with an incentive to post mechanisms that decentralize the planner's solution, which requires efficiency along two margins: (i) the allocation of buyers to

[41] See the online appendix for a detailed derivation.

sellers, and (ii) the allocation of the good given a queue of buyers. The second-price auction fulfills the second requirement and provides each buyer with a payoff equal to the extra surplus that he creates when he has the highest valuation. To satisfy the first requirement however, each buyer must receive an expected payoff exactly equal to his marginal contribution to social surplus, which includes the externality that he may impose during the meeting process (e.g. by preventing a buyer with a higher valuation from meeting the seller). The meeting fees allow this externality to be priced.[42] The meeting fee captures the positive or negative spillovers that buyers impose on each other (the numerator) conditional on the event that a buyer meets a seller (denominator, the term $\phi_\mu(0,\Lambda)$ is equal to the probability that a buyer meets a seller). The reason that the spillovers can be priced with a single fee/subsidy is that buyers are only heterogeneous with respect to their valuation and not in terms of their search intensities.

3.4 Virtual valuation and ϕ

In a standard second-price auction with n bidders with a reserve price, r, the seller's expected profit is

$$\pi_n = \int_r^1 \left(x - \frac{1-F(x)}{f(x)} \right) dF^n(x), \tag{3.3}$$

where $x - \frac{1-F(x)}{f(x)}$ is the virtual value function, see Myerson (1981).

To illustrate the relationship between the virtual valuation function and $\phi(\lambda F(dx),\lambda)$ in a competing mechanism environment, we first focus for simplicity on the invariant case where $r = 0$ and no fees or subsidies are charged.

[42]The equilibrium is of course not unique, as a seller could replace the second-price by a first-price auction (because of revenue equivalence) or double the meeting fee but charging it with probability $\frac{1}{2}$ (because of risk-neutrality). However, all equilibrium mechanisms are payoff-equivalent. See e.g. Peters and Severinov (1997), Albrecht et al. (2014), Lester et al. (2015) for detailed discussions of efficiency in related models.

The seller will receive π_n in equation (3.3) with probability $P_n(\lambda)$. Therefore, for a given λ, the expected profit of a seller is

$$\pi = \sum_0^\infty P_n(\lambda)\pi_n = \int_0^1 \left(x - \frac{1-F(x)}{f(x)}\right) \sum_0^\infty P_n(\lambda) dF^n(x)$$
$$= \int_0^1 (x - \frac{1-F(x)}{f(x)}) d \sum_0^\infty P_n(\lambda) F^n(x)$$
$$= \int_0^1 \left(x - \frac{1-F(x)}{f(x)}\right) d\big(1 - \phi(\lambda(1-F(x)), \lambda)\big),$$

where in the first line we interchange integration and summation. In a standard auction with n bidders, the buyer with the highest valuation, x, pays to the seller(in the terminology of Bulow and Roberts (1989)) the lowest value he could have announced without losing the auction. This equals the expected value of the second highest bid. So the expected payoff of the seller equals the virtual valuation integrated against the distribution of highest valuations out of n buyers. The latter is simply $F^n(x)$. In our setting, the probability that x is the highest valuation will depend on the meeting technology. The term $\big(1 - \phi(\lambda(1-F(x)), \lambda)\big)$ gives the probability that there are no buyers with valuations above x. For the urn ball meeting technology where $\phi(\mu, \lambda) = 1 - e^{-\mu}$, this just boils down to

$$\pi = \int_0^1 (x - \frac{1-F(x)}{f(x)}) d e^{-\lambda(1-F(x))}$$
$$= \lambda \int_0^1 (x - \frac{1-F(x)}{f(x)}) e^{-\lambda(1-F(x))} f(x) dx.$$

Those expressions can easily be adjusted for general meeting technologies which may include a separate entrance fee or subsidy that prices any spillovers that buyers impose on each other. One may have expected that allowing for general meeting technologies, would severely complicate the payoff functions in (competing) auction theory. We have shown here that this is not the case because we can just replace the distribution of highest valuations by $\big(1-\phi(\lambda(1-F(x)), \lambda)\big)$.

3.5 Classification of meeting technologies

Bilateral meetings are well understood, but love for variety is a novel condition and warrants discussion. To better understand this condition, we compare it in this section to two other properties of meeting technologies described in the literature, *invariance* and *non-rivalry*. We show that invariance is a sufficient (but not a necessary) condition, while non-rivalry is a necessary (but not a sufficient) condition. Figure 8 summarizes this discussion.

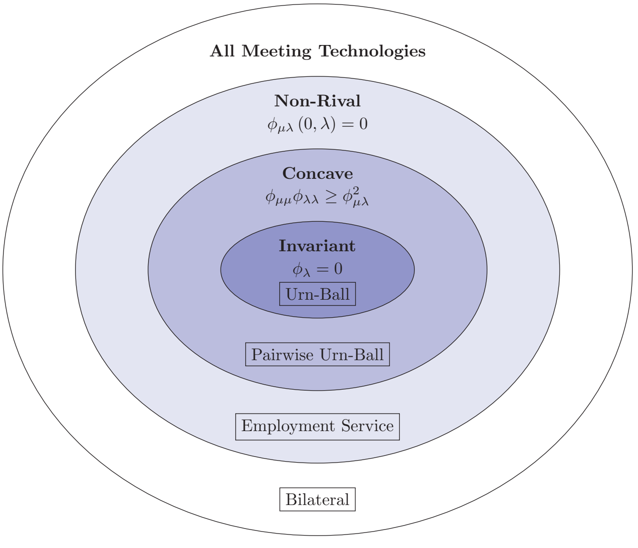

Figure 8: Classification of Meeting Technologies

Invariance. Introduced by Lester et al. (2015), an invariant technology is one in which the queue of blue buyers μ at a seller is a sufficient statistic for the distribution of the number of meetings between blue buyers and that seller. Formally,

$$\sum_{N=n}^{\infty} P_N(\lambda) \binom{N}{n} \left(\frac{\mu}{\lambda}\right)^n \left(1 - \frac{\mu}{\lambda}\right)^{N-n} = P_n(\mu), \tag{3.4}$$

for all $0 \leq \mu \leq \lambda < \infty$ and $n \in \mathbb{N}_0$. We first establish that if (3.4) holds for $n = 0$, then it holds for all n. That is, invariance can alternatively be defined as the condition that the probability that a seller meets *at least one* of the μ blue buyers is independent of the number of other buyers visiting the same submarket, as formalized by the following lemma.

Lemma 9. *A meeting technology is invariant if and only if* $\phi_\lambda(\mu,\lambda) = 0$ *for all* $0 \le \mu \le \lambda < \infty$.

Perhaps the best-known example of an invariant technology is the urn-ball technology.[43]. As shown by e.g. McAfee (1993), Peters and Severinov (1997) and Albrecht et al. (2014), this technology leads to an equilibrium with a single market in which all agents participate.[44] The following lemma extends this result to all invariant technologies and establishes that while invariance is a sufficient condition for love for variety, it is not a necessary condition.

Proposition 10. *Invariance implies love for variety, but love for variety does not imply invariance.*

It is straightforward to see why invariance is sufficient. Invariance implies that the presence of low-type buyers in the submarket has no effect on the meetings between high-type buyers and sellers. Surplus is therefore maximized by spreading high-type buyers evenly across all sellers, as opposed to concentrating them at a subset, in order to maximize the number of high-type buyers that will trade. A single market results.

To understand why invariance is not necessary, consider the pairwise urn-ball technology. As explained by Lester et al. (2015), this technology is not invariant. Intuitively, when there are very few low-type buyers in the submarket, most buyer pairs consist of two high types, making it likely that a seller will meet an even number of buyers with high valuations. Adding additional low-type buyers to this submarket increases the probability that a buyer pair will consist of one low and one high type, and that a seller will meet an odd number of high-type buyers. This makes it more likely that a seller will meet at least one high-type buyer, i.e. $\phi_\lambda > 0$. This feature violates invariance, but not love for variety: the fact that the addition of low-type buyers to the submarket helps to spread the high-type buyers better across sellers strengthens the incentive to send all buyers to the same market.[45]

[43] Urn-ball gives $\phi(\mu,\lambda) = 1 - e^{-\mu}$. As discussed in Lester et al. (2015), a second example of an invariant technology is the geometric distribution $P_n(\lambda) = \frac{\lambda^n}{(1+\lambda)^{n+1}}$, which yields $\phi(\mu,\lambda) = \frac{\mu}{1+\mu}$.

[44] Shi (2006) derives a similar result in a labor-market setting.

[45] This may raise the question how $\phi_\lambda \ge 0$ relates to love for variety. We prove in the online appendix that it is a necessary but not a sufficient condition.

Non-Rivalry. Eeckhout and Kircher (2010b) define a (purely) non-rival technology as one in which the probability for a buyer to meet a seller is not affected by the presence of other buyers in the market. We first establish that their definition is equivalent to $\phi_{\mu\lambda}(0,\lambda) = 0$ for all $0 \leq \lambda < \infty$.

Lemma 11. *A meeting technology is non-rival if and only if* $\phi_{\mu\lambda}(0,\lambda) = 0$ *for all* $0 \leq \lambda < \infty$.

To understand this expression, recall that $\phi(\mu,\lambda)$ represents the probability that a seller meets at least one blue buyer, which is clearly zero if $\mu = 0$. The partial derivative $\phi_{\mu}(0,\lambda)$ captures how this changes if a single buyer (or more precisely, an arbitrarily small measure of buyers) in the queue becomes blue and must therefore equal the probability that this blue buyer succeeds in meeting the seller. Since meetings are type-independent, the same expression applies to all λ buyers in the queue, irrespective of how many of them are blue. Non-rivalry then says that this meeting probability should be independent of λ.

It is easy to verify that the above examples of technologies that exhibit love for variety, i.e. urn-ball and pairwise urn-ball, both satisfy non-rivalry. This is not a coincidence. As the following proposition establishes, all technologies that exhibit love for variety are non-rival. However, not all non-rival technologies exhibit love for variety.

Proposition 12. *Love for variety implies non-rivalry, but non-rivalry does not imply love for variety.*

To understand why non-rivalry is a necessary condition for love for variety, consider a submarket with a single high-type buyer with valuation $x_2 > 0$ and a number of low-type buyers with valuation $x_1 \to 0$, such that surplus only depends on the trading probability of the high-type buyer. Violation of non-rivalry would imply that this probability could be increased by sending either some low-type buyers (if $\phi_{\mu\lambda}(0,\lambda) < 0$) or some sellers (if $\phi_{\mu\lambda}(0,\lambda) > 0$) to a different submarket, contradicting the optimality of the single market associated with love for variety.

To see why non-rivalry is not sufficient, consider the multi-platform technology. Clearly, every buyer meets a seller with probability 1, which means that this technology is non-rival. However, the presence of low-type buyers in the submarket increases the chances for high-type buyers to be crowded out at one of the αs sellers in the first round, concentrating them at the $(1-\alpha)\,s$ second-round sellers in higher numbers than optimal. It is therefore better to send at least some low types to a separate submarket. Hence, non-rivalry does not imply love for variety.[46]

3.6 Conclusion

We study an environment in which sellers compete for heterogeneous buyers by posting mechanisms. Buyers can direct their search to the mechanism that maximizes their expected payoff, but may experience frictions in meeting a particular seller. We derive necessary and sufficient conditions on the technology that governs these meetings under which either perfect separation (a separate submarket for each type of buyer) or perfect pooling (a single market in which all buyers participate) are optimal. We find that perfect separation is the efficient outcome if and only if meetings are bilateral, while perfect pooling arises if the meeting technology satisfies a novel property, which we call "love for variety."

Proofs

Proof of Lemma 3. The maximum valuation at a seller who meets $n \in \mathbb{N}_1$ buyers is an order statistic, distributed according to $G^n(x)$. Taking the expectation over x and n, followed by integration by parts and using the Dominated Convergence Theorem to interchange summation and integration, yields

$$S(\lambda, G) = \sum_{n=1}^{\infty} P_n(\lambda) \int_0^1 x\, dG^n(x) = \int_0^1 \left(1 - \sum_{n=0}^{\infty} P_n(\lambda)\, G^n(x)\right) dx.$$

The result then follows because the rightmost integrand equals $\phi(\lambda(1-G(x)), \lambda)$.

□

[46]This contradicts proposition 5 in Eeckhout and Kircher (2010b) which states that non-rivalry is a sufficient condition for a single market. The discrepancy originates in the fact that the proof of their proposition implicitly assumes invariance rather than non-rivalry when treating the trading probability for high-type buyers as independent of the queue of low-type buyers.

Proof of Proposition 4. **Part 1** (bilateral meetings imply full separation): Suppose that there exists a submarket with a measure s of sellers and a queue (λ, G). Because of lemma 3 and the fact that meetings are bilateral, the surplus created in this submarket equals

$$sS(\lambda, G) = sP_1(\lambda)\int_0^1 (1 - G(x))\,dx. \tag{.5}$$

Now suppose the planner would decompose this submarket into a separate submarket for each type of buyer, allocating sellers in such a way that the queue length in each new submarket remains λ. A seller in the submarket for type x then creates a surplus $P_1(\lambda)\,x$, such that surplus across all submarkets equals

$$s\int_0^1 P_1(\lambda)\,x\,dG(x). \tag{.6}$$

Clearly, (.5) and (.6) are equal to each other. The result then follows because the allocation of sellers in (.6) is suboptimal—the planner can increase surplus by allocating more sellers to the submarkets in which buyers have high valuations than to the submarkets where they have low valuations.

Part 2 (full separation implies bilateral meetings): We prove this result by showing that if meetings are not bilateral for some $\Lambda > 0$, i.e. $P_0(\Lambda) + P_1(\Lambda) < 1$, then there exists a two-type distribution of buyers such that full separation is not optimal.[47] To do so, suppose the market is populated by a measure 1 of sellers, a measure b_1 of buyers with valuation x_1, and a measure b_2 of buyers with valuation x_2, satisfying $b_1 + b_2 = \Lambda$ and $x_2 > x_1$.

Suppose the planner fully separates the two types of buyers and optimally allocates s_i sellers to the submarket for valuation x_i, where $s_1 + s_2 = 1$. Define queue lengths $\lambda_i = \frac{b_i}{s_i}$. Clearly, if $x_1 \to x_2$, then $s_1 \to \frac{b_1}{b_1+b_2}$ and $s_2 \to \frac{b_2}{b_1+b_2}$, which implies $\lambda_1 \to \Lambda$ and $\lambda_2 \to \Lambda$.

Let now a measure ε of buyers with valuation x_1 and an equally large measure of buyers with valuation x_2 swap submarket, such that—in both submarkets—the queue lengths stay the same, but the composition of types

[47] Of course, if $P_0(\Lambda) + P_1(\Lambda) < 1$, then—by continuity—there exists a small neighborhood of Λ for which $P_0 + P_1 < 1$.

becomes marginally more diverse. Again by lemma 3, social surplus of this new allocation equals

$$\begin{aligned}\mathscr{S}(\varepsilon) &= s_2\left[(x_2-x_1)\phi\left(\frac{b_2-\varepsilon}{s_2},\lambda_2\right)+x_1\phi(\lambda_2,\lambda_2)\right]\\ &\quad +s_1\left[(x_2-x_1)\phi\left(\frac{\varepsilon}{s_1},\lambda_1\right)+x_1\phi(\lambda_1,\lambda_1)\right].\end{aligned}$$

Clearly, $\varepsilon = 0$ corresponds to full separation. To analyze whether this is the optimal outcome, consider the change in surplus associated with a marginal increase in ε, i.e.

$$\mathscr{S}'(0) = (x_2-x_1)\left(\phi_\mu(0,\lambda_1)-\phi_\mu(\lambda_2,\lambda_2)\right) \tag{.7}$$

Note that

$$\phi_\mu(\mu,\lambda) = \frac{P_1(\lambda)}{\lambda}+\frac{1}{\lambda}\sum_{n=2}^{\infty} nP_n(\lambda)\left(1-\frac{\mu}{\lambda}\right)^{n-1},$$

which implies that $\phi_\mu(0,\lambda) \geq \frac{P_1(\lambda)}{\lambda}+\frac{2(1-P_0(\lambda)-P_1(\lambda))}{\lambda}$ and $\phi_\mu(\lambda,\lambda) = \frac{P_1(\lambda)}{\lambda}$. Substitution into (.7) yields

$$\frac{1}{x_2-x_1}\mathscr{S}'(0) \geq \frac{P_1(\lambda_1)}{\lambda_1}+\frac{2(1-P_0(\lambda_1)-P_1(\lambda_1))}{\lambda_1}-\frac{P_1(\lambda_2)}{\lambda_2}.$$

Let $x_1 \to x_2$, such that $\lambda_1 \to \Lambda$ and $\lambda_2 \to \Lambda$. Then

$$\frac{1}{x_2-x_1}\mathscr{S}'(0) \to \frac{2(1-P_0(\Lambda)-P_1(\Lambda))}{\Lambda} > 0.$$

Hence, full separation is not optimal and social surplus can be increased by slightly mixing the submarkets. □

Proof of Proposition 6. **Part 1** (love for variety implies perfect pooling): To prove this result, suppose that there are two submarkets, indexed by $i \in \{1,2\}$, consisting of $s_i > 0$ sellers who each have a queue (λ_i, G_i). By lemma 3, total surplus across the two submarkets is equal to

$$s_1\int_0^1 \phi(\lambda_1(1-G_1(x)),\lambda_1)\,dx + s_2\int_0^1 \phi(\lambda_2(1-G_2(x)),\lambda_2)\,dx. \tag{.8}$$

We show a higher surplus can be generated by merging the two submarkets, creating one market with $s_0 = s_1+s_2$ sellers, each with a queue $\lambda_0 = \frac{s_1\lambda_1+s_2\lambda_2}{s_1+s_2}$ of buyers whose valuations are distributed according to

$$G_0(x) = \frac{s_1\lambda_1G_1(x)+s_2\lambda_2G_2(x)}{s_1\lambda_1+s_2\lambda_2}.$$

Again by lemma 3, this combined market will create a surplus $s_0 \int_0^1 \phi\left(\lambda_0\left(1-G_0(x)\right), \lambda_0\right) dx$, which is larger than (.8) because concavity of $\phi\left(\mu, \lambda\right)$ implies that

$$s_1 \phi\left(\mu_1, \lambda_1\right) + s_2 \phi\left(\mu_2, \lambda_2\right) \leq s_0 \phi\left(\frac{s_1\mu_1 + s_2\mu_2}{s_1 + s_2}, \frac{s_1\lambda_1 + s_2\lambda_2}{s_1 + s_2}\right).$$

Hence, a single market is optimal for technologies that exhibit love for variety.

Part 2 (perfect pooling implies love for variety): We prove this result by showing that if ϕ is not concave, there exists a two-type distribution of buyers such that one market is not optimal. Note that if ϕ is not concave, then—by the definition of concavity—there exist values α, μ_1, μ_2, λ_1 and λ_2, such that

$$\alpha\phi(\mu_1, \lambda_1) + (1-\alpha)\phi(\mu_2, \lambda_2) > \phi(\mu_0, \lambda_0), \tag{.9}$$

where $\mu_0 = \alpha\mu_1 + (1-\alpha)\mu_2$ and $\lambda_0 = \alpha\lambda_1 + (1-\alpha)\lambda_2$.

Consider now a market in which buyers' valuations are either x_1 or x_2, with $0 < x_1 < x_2$. Set the measure of high-type buyers equal to μ_0 and the measure of low-type buyers equal to $\lambda_0 - \mu_0$, while maintaining the assumption that the measure of sellers equals 1. Then by Lemma 3, the social surplus of creating a single market is $\mathscr{S}_1 = (x_2 - x_1)\phi(\mu_0, \lambda_0) + x_1\phi(\lambda_0, \lambda_0)$.

Now, decompose the single market into two submarkets A and B, with seller measures α and $1-\alpha$, total queue lengths λ_1 and λ_2, and high-type queue lengths μ_1 and μ_2, respectively.[48] The social surplus per seller for the two submarkets is

$$\begin{aligned} S_2^A &= (x_2 - x_1)\phi(\mu_1, \lambda_1) + x_1\phi(\lambda_1, \lambda_1) \\ S_2^B &= (x_2 - x_1)\phi(\mu_2, \lambda_2) + x_1\phi(\lambda_2, \lambda_2) \end{aligned}$$

and total surplus across the two submarkets equals $\mathscr{S}_2 = \alpha S_2^A + (1-\alpha)S_2^B$.

In the limit $x_1 \to 0$, the two submarkets create more surplus than the single market, i.e. $\mathscr{S}_2 > \mathscr{S}_1$, if and only if

$$x_2\left(\alpha\phi(\mu_1, \lambda_1) + (1-\alpha)\phi(\mu_2, \lambda_2)\right) > x_2\phi(\mu_0, \lambda_0),$$

which holds because it is exactly equation (.9). Hence, love for variety is a necessary condition for a single market. □

[48] This is possible because $\mu_0 = \alpha\mu_1 + (1-\alpha)\mu_2$ and $\lambda_0 = \alpha\lambda_1 + (1-\alpha)\lambda_2$.

Proof of Proposition 8. Consider a seller who posts a second-price auction and a meeting fee τ, attracting a queue (λ, G). A buyer with valuation x meets the seller together with $n-1$ other buyers with probability $\frac{nP_n(\lambda)}{\lambda}$.[49] Hence, he pays the meeting fee τ with probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda) = \phi_\mu(0,\lambda)$ and trades with probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda)\, G(x)^{n-1} = \phi_\mu(\lambda(1-G(x)),\lambda)$. As a result, his expected payoff is

$$U(x,\tau,\lambda,G) = -\phi_\mu(0,\lambda)\,\tau + \int_0^x \phi_\mu\left(\lambda\left(1-G\left(y\right)\right),\lambda\right) dy, \tag{.10}$$

where the second term is the payoff from the auction, which—by standard results in auction theory—equals the integral over the trading probabilities (see e.g. Peters, 2013). The payoff for the seller is the difference between surplus and buyers' payoffs, i.e.

$$R(\tau,\lambda,G) = \int_0^1 \phi\left(\lambda\left(1-G\left(y\right)\right),\lambda\right) dy - \lambda \int_0^1 \overline{U}\left(y\right) dG\left(y\right). \tag{.11}$$

In the proposed equilibrium, $\tau = -\frac{\int_0^1 \phi_\lambda(\Lambda(1-F(x)),\Lambda)\,dx}{\phi_\mu(0,\Lambda)}$, $\lambda = \Lambda$, and $G = F$. Evaluating (.10) and (.11) in these values yields the equilibrium payoffs for buyers and sellers.

Next, we establish that no seller would like to deviate by showing that even if a seller can choose any queue composition directly, he could not do better than the proposed equilibrium mechanism. In this relaxed maximization problem, the queue of buyers of type x will be such that their marginal contribution to social surplus is equal to their marginal cost $\overline{U}(x)$. Since a seller's payoff R is concave in the queue function $\lambda G(x)$ (an infinite dimensional object), the first-order conditions are also sufficient for optimality. With a queue composition $\Lambda F(x)$, the marginal contribution to social surplus of a buyer of type x is

$$V(x) = \int_0^1 \phi_\lambda\left(\Lambda\left(1-F\left(y\right)\right),\Lambda\right) dy + \int_0^x \phi_\mu\left(\Lambda\left(1-F\left(y\right)\right),\Lambda\right) dy.$$

where the first term captures the externalities the buyer imposes on the other agents by changing the total queue length, and the second term is the buyer's direct contribution to social surplus when he is the buyer with the highest valuation in the auction. Since $\overline{U}(x) = V(x)$ under the proposed equilibrium, the solution to the relaxed maximization is (Λ, F) and the seller can indeed not increase his payoffs by deviating from the proposed equilibrium. □

[49] See Eeckhout and Kircher (2010b) or Lester et al. (2015).

Proof of Lemma 9. **Part 1** (invariance implies $\phi_\lambda = 0$): Evaluating the definition of invariance (3.4) in $n = 0$ yields

$$\sum_{N=0}^{\infty} P_N(\lambda)\left(1-\frac{\mu}{\lambda}\right)^N = P_0(\mu). \tag{.12}$$

The left-hand side of this equation is $1-\phi(\mu,\lambda)$ and the right-hand side is independent of λ. Hence, $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \le \mu \le \lambda < \infty$.

Part 2 ($\phi_\lambda = 0$ implies invariance): Note that $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \le \mu \le \lambda < \infty$ implies that $\phi(\mu,\lambda) = \phi(\mu,\mu)$. By equation (4.1), $\phi(\mu,\mu) = 1 - P_0(\mu)$. Consequently, equation (.12) must hold for all $0 \le \mu \le \lambda < \infty$. By standard results from analytic function theory (see e.g. Ahlfors, 1979, p.32), we can differentiate both sides of this equation n times with respect to μ, which yields

$$\sum_{N=n}^{\infty} \frac{N!}{(N-n)!} P_N(\lambda)\left(-\frac{1}{\lambda}\right)^n\left(1-\frac{\mu}{\lambda}\right)^{N-n} = P_0^{(n)}(\mu), \tag{.13}$$

for all $0 \le \mu \le \lambda < \infty$. For $\mu = \lambda$, this gives $P_0^{(n)}(\mu) = \frac{n!}{(-\mu)^n} P_n(\mu)$. Substitute this into the right hand side of equation (.13) and rearrange the term $\frac{n!}{(-\mu)^n}$ to the left hand side give (3.4). Hence, $\phi_\lambda = 0$ implies invariance. □

Proof of Proposition 10. **Part 1** (invariance implies love for variety): This result follows immediately from lemma 9: $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \le \mu \le \lambda < \infty$ implies that $\phi_{\lambda\lambda}(\mu,\lambda) = \phi_{\mu\lambda}(\mu,\lambda) = 0$ for all $0 \le \mu \le \lambda < \infty$, which in turn implies that equation (3.2) is satisfied.

Part 2 (love for variety does not imply invariance): Consider the pairwise urn-ball technology, which satisfies $\phi(\mu,\lambda) = 1 - e^{-\mu(1-\frac{1}{2}\frac{\mu}{\lambda})}$. Since $1-e^{-y}$ is an increasing, concave function, a sufficient condition for $\phi(\mu,\lambda)$ to be concave is that the map $(\mu,\lambda) \to \mu(1-\frac{1}{2}\frac{\mu}{\lambda})$ is concave.[50] The Hessian of this map is indeed negative semi-definite. However, the technology is not invariant, as

$$\phi_\lambda(\mu,\lambda) = \frac{1}{2}\frac{\mu^2}{\lambda^2} e^{-\mu(1-\frac{1}{2}\frac{\mu}{\lambda})} > 0.$$

Hence, love for variety does not imply invariance. □

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[50] See, for example, Theorem 5.1 in Rockafellar (1970, p.32).

Proof of Lemma 11. As shown by Lester et al. (2015), non-rivalry is satisfied if and only if $\frac{\partial}{\partial \lambda}\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda) = 0$, for all $0 \leq \lambda < \infty$. The desired result then follows directly from observing that $\phi_\mu(0,\lambda) = \frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda)$. □

Proof of Proposition 12. **Part 1** (love for variety implies non-rivalry): Suppose a technology does not satisfy non-rivalry, i.e. $\phi_{\mu\lambda}(0,\lambda) \neq 0$. Then

$$\phi_{\mu\mu}(0,\lambda)\,\phi_{\lambda\lambda}(0,\lambda) - \phi_{\mu\lambda}^2(0,\lambda) < \phi_{\mu\mu}(0,\lambda)\,\phi_{\lambda\lambda}(0,\lambda) = 0,$$

since $\phi(0,\lambda) = 0$ for all $0 \leq \lambda < \infty$. In words, ϕ is not concave. Hence, love for variety implies non-rivalry.

Part 2 (non-rivalry does not imply love for variety): Consider the multi-platform technology. Starting from the expression for $\phi(\mu,\lambda)$ for this technology, one can derive

$$\phi_{\mu\lambda} = -\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}} - \frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)} e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right)\frac{\alpha}{(\lambda+\alpha)^2} \leq 0,$$

which equals 0 (only) for $\mu = 0$. Hence, the multi-platform technology is non-rival.

Further, we get

$$\phi_\lambda = -\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right)\frac{\mu\alpha}{(\lambda+\alpha)^2},$$

which is strictly negative for all $0 < \mu \leq \lambda < \infty$ and $\alpha > 0$. As we show in the online appendix, $\phi_\lambda \geq 0$ is a necessary condition for love for variety. Hence, the multi-platform technology does not exhibit this property. □

Decentralized Equilibrium in the Bilateral Case

We first solve the planner's problem and then show that it can be implemented as a decentralized equilibrium. For analytic convenience we assume that the buyer value distribution F admits a density function f but our conclusion holds for general F as well.

With bilateral meeting technologies, the planner chooses perfect segmentation. Therefore, we can index submarkets by buyer value x. The planner assigns a measure of sellers, in submarket x, $s(x)$ and a queue length $\lambda(x) = \frac{\Lambda f(x)}{s(x)}$ for this submarket. The social planner's problem is to choose $s(x)$ to maximize

$$\int_0^1 s(x) x P_1(\frac{\Lambda F(x)}{s(x)}) dx$$

subject to $\int_0^1 s(x) dx = 1$. The first order condition with respect to $s(x)$ conditional on x being active in the market, i.e., $s(x) > 0$ (note that low x buyer types can be excluded, resulting in $s(x) = 0$)

$$x(P_1(\lambda(x)) - \lambda(x) P_1'(\lambda(x))) = \pi \qquad (.14)$$

where π is the marginal contribution of sellers in market x. Since P_1 is concave, the first order conditions are also sufficient for optimality.

Define $R(\lambda) = P_1(\lambda) - \lambda P_1'(\lambda)$. $R(\lambda)$ is strictly increasing since P_1 is strictly concave. Equation (.14) can be rewritten as $xR(x) = \pi$ for $s(x) > 0$, which implies if $x_1 < x_2$, $\lambda(x_1) > \lambda(x_2)$. Further note that $R(0) = 0$ and $R(\infty) = 1$.[51] Then the lowest active type must be $\underline{x} = \pi$. Example: $P_1(\lambda) = 1 - e^{-\lambda} \Rightarrow R(\lambda) = 1 - e^{-\lambda} - \lambda e^{-\lambda}$.

As a result, the planner's problem is solved by the value of π such that

$$\int_\pi^1 \frac{\Lambda f(x)}{R^{-1}(\frac{\pi}{x})} dx = 1, \qquad (.15)$$

where the RHS is the total measure of sellers. Note the LHS is a decreasing function of π.

Next we show that the planner's solution can be decentralized. We will verify that in submarket x, sellers post $\tau(x) = \pi / P_1(\lambda(x))$ is an equilibrium, where π is given by equation (.15) and satisfies equation (.14).

[51] Here we will prove $R(\infty) = 1$. First note $R(\lambda) < 1$ and $R'(\lambda) = -\lambda P_1''(\lambda) > 0$. Thus $R(\lambda) \uparrow a$ as $\lambda \to \infty$. Since $P_1(\lambda) \uparrow 1$, $\lambda P_1'(\lambda) \to 1 - a$ as $\lambda \to \infty$. If $a < 1$, then there exists N such that for $\lambda > N$, $\lambda P_1'(\lambda) > (1-a)/2$, i.e., $P_1'(\lambda) > \frac{1-a}{2\lambda}$. As a result, $1 - P_1(N) = \int_N^\infty P_1'(\lambda) d\lambda > \int_N^\infty \frac{1-a}{2\lambda} d\lambda = \frac{1-a}{2}(\log(\infty) - \log(N)) = \infty$. Contradiction arises.

Step 1: we need to show that buyers x do not go to submarkets other than x. That is,

$$(x-\tau(x))Q_1(\lambda(x)) > (x-\tau(y))Q_1(\lambda(y))$$

Use $x=\pi/R(x)$ and $\tau(x)=\pi/P_1(\lambda(x))$, the above is equivalent to

$$\frac{P_1(\lambda(x))}{R(\lambda(x))\lambda(x)}-\frac{1}{\lambda(x)} > \frac{P_1(\lambda(y))}{R(\lambda(x))\lambda(y)}-\frac{1}{\lambda(y)}$$

which is equivalent to the requirement that the right hand side achieves a maximum at $y=x$. Differentiating the right hand side with respect to $\lambda(y)$ (not y) gives,

$$\frac{R(\lambda(x))-R(\lambda(y))}{R(\lambda(x))\lambda^2(y)}.$$

Thus the optimum is $\lambda(y)=\lambda(x)$, i.e., $y=x$. As a result, buyers choose to participate in their own market.

Step 2: we need to prove that sellers have no incentive to deviate from the equilibrium. Since posted prices are in $[\pi,\tau(\overline{x})]$, $\overline{x}=1$, and generate the same profit, the deviation has to be either smaller than π or bigger than $\tau(\overline{x})$. Since the expected profit is already π, prices smaller than π is not an option. When posting a price $\widetilde{\tau}$ higher than $\tau(\overline{x})$, the question facing a seller is what queue composition he will attract. Following Eeckhout and Kircher (2010a,b), we assume that the seller will expect the longest queue (this gives the strongest incentive to deviate). The longest queue is one where all buyers who visit the deviant have value $\overline{x}$, and the queue length $\widetilde{\lambda}$ is such that $(\overline{x}-\tau(\overline{x}))Q_1(\lambda(\overline{x}))=(\overline{x}-\widetilde{\tau})Q_1(\widetilde{\tau})$. By the accounting identity of the surplus $f(\overline{x})U(\overline{x})+s(\overline{x})\pi=S(f(\overline{x}),s(\overline{x}))$ (where S is the total surplus in submarket $\overline{x}$ and is homogeneous of degree 1), a potential deviant seller will choose $\widetilde{\tau}$ and consequently $\widetilde{\lambda}$ in order to maximize $\overline{x}P_1(\widetilde{\lambda})-\widetilde{\lambda}U(\overline{x})$. Since the partial derivative $S_2=\pi$ and $S=f(x)S_1+s(x)S_2$, thus the partial derivative $S_1=\overline{x}P_1(\lambda(\overline{x}))=U(\overline{x})$. As a result, sellers' profit is already maximized at $\widetilde{\tau}=\tau(\overline{x})$, and there is no profitable deviation for sellers.

Entry of Sellers

Proposition 13. *When the meeting technology is bilateral or exhibits love for variety, the market equilibrium with free entry is efficient.*

Proof. Bilateral case: We have already shown that $\frac{\partial S(n_s, n_b F)}{\partial n_s} = \pi$ for any n_s. Efficient entry follows directly.

Love for variety case: to simplify notation, we assume buyer types are finite. Buyers have values $x_1 < x_2 < \cdots < x_n$, with total measure $b_1, b_2, \ldots, b_n$ correspondingly. Denote the social surplus as $S(n_s, b_1, \ldots, b_n)$. We have shown in Proposition 8, $U(x_i) = \frac{\partial S(n_s, b_1, \ldots, b_n)}{\partial b_i}$. Note

$$
\begin{aligned}
S(n_s, b_1, \ldots, b_n) &= n_s \pi + \sum_1^n b_i U(x_i) \\
&= n_s \frac{\partial S(n_s, b_1, \ldots, b_n)}{\partial n_s} + \sum_1^n b_i \frac{\partial S(n_s, b_1, \ldots, b_n)}{\partial b_i}
\end{aligned}
$$

where the first equality follows from an accounting identity and the second follows from the fact that because the function S is homogeneous of degree 1. As a result, $\pi = \frac{\partial S(n_s, b_1, \ldots, b_n)}{\partial n_s}$, and seller entry is efficient again. □

Love for Variety Using P_n

In the main text, we define love for variety in terms of ϕ, but an equivalent condition in terms of P_n, the actual primitive of the model, can be derived.[52] Starting from the definition (4.1), taking partial derivatives of ϕ yields

$$
\begin{aligned}
\phi_{\mu\mu} &= -\sum_{n=0}^{\infty} (n+2)(n+1) \frac{P_{n+2}}{\lambda^2} \left(1 - \frac{\mu}{\lambda}\right)^n, \\
\phi_{\mu\lambda} &= \sum_{n=0}^{\infty} \left[(n+1) \frac{\lambda P'_{n+1} - P_{n+1}}{\lambda^2} + (n+2)(n+1) P_{n+2} \frac{\mu}{\lambda^3} \right] \left(1 - \frac{\mu}{\lambda}\right)^n, \\
\phi_{\lambda\lambda} &= -\sum_{n=0}^{\infty} \left[P''_n + 2\mu(n+1) \frac{\lambda P'_{n+1} - P_{n+1}}{\lambda^3} + \frac{(n+2)(n+1) P_{n+2} \mu^2}{\lambda^4} \right] \left(1 - \frac{\mu}{\lambda}\right)^n.
\end{aligned}
$$

Using the fact that $\sum_{n=0}^{\infty} a_n y^n \sum_{n=0}^{\infty} b_n y^n - \left(\sum_{n=0}^{\infty} c_n y^n\right)^2 = \sum_{n=0}^{\infty} \sum_{i=0}^{n} \left(a_i b_{n-i} - c_i c_{n-i}\right) y^n$, condition (3.2) can then be written as

$$
\phi_{11}\phi_{22} - \phi_{12}^2 = \sum_{n=0}^{\infty} Z_n \left(1 - \frac{\mu}{\lambda}\right)^n \geq 0,
$$

where, after some simplification, Z_n equals

$$
Z_n = \sum_{i=0}^{n} \left[\frac{(i+2)(i+1) P_{i+2} P''_{n-i}}{\lambda^2} - (i+1)(n-i+1) \frac{\lambda P'_{i+1} - P_{i+1}}{\lambda^2} \frac{\lambda P'_{n-i+1} - P_{n-i+1}}{\lambda^2} \right].
$$

[52] To save on notation, we suppress the argument of P_n throughout this derivation.

Necessity and Insufficiency of $\phi_\lambda \geq 0$

The fact that the pairwise urn-ball technology satisfies $\phi_\lambda \geq 0$ as well as love for variety may raise the question how these two properties are related. The following proposition establishes that that $\phi_\lambda(\mu, \lambda) \geq 0$ for all $0 \leq \mu \leq \lambda < \infty$ is a necessary but not a sufficient condition for love for variety.

Proposition 14. *Love for variety implies $\phi_\lambda \geq 0$, but $\phi_\lambda \geq 0$ does not imply love for variety.*

Proof. Part 1 (love for variety implies $\phi_\lambda \geq 0$): We prove this result by contradiction. Suppose that there exists a meeting technology for which $\phi(\mu, \lambda)$ is concave in (μ, λ), but $\phi_\lambda(\mu_0, \lambda_0) < 0$ in some point (μ_0, λ_0). Note that $\phi_{\mu\mu} < 0$ for all technologies that exhibit love for variety, hence $\phi(\mu, \lambda)$ must also be concave in λ alone, i.e. $\phi_{\lambda\lambda} \leq 0$. In other words, $\phi_\lambda(\mu, \lambda)$ is a non-increasing function of λ, such that $\phi_\lambda(\mu_0, \lambda) \leq \phi_\lambda(\mu_0, \lambda_0) < 0$ for all $\lambda > \lambda_0$. This implies that $\phi(\mu_0, \lambda) \leq \phi(\mu_0, \lambda_0) + \phi_\lambda(\mu_0, \lambda_0)(\lambda - \lambda_0)$ for all $\lambda > \lambda_0$. Let $\lambda \to \infty$ and thus $\phi_\lambda(\mu_0, \lambda_0)(\lambda - \lambda_0) \to -\infty$, such that $\phi(\mu_0, \lambda) \to -\infty$. Since ϕ is a probability, this leads to the required contradiction. Hence, concavity of $\phi(\mu, \lambda)$, i.e. love for variety, implies $\phi_\lambda \geq 0$.

Part 2 ($\phi_\lambda \geq 0$ does not imply love for variety): Consider the following meeting technology.

> *Minimum Demand.* This technology consists of two rounds. In the first round, the b buyers in the submarket are allocated to the s sellers according to the urn-ball technology. In the second round, each seller draws a minimum demand requirement and operates only if the number of buyers that came to him weakly exceeds this minimum.[53] We assume that the minimum demand requirements follow a geometric distribution, such that the minimum is weakly less than $n \in \mathbb{N}_1$ with probability $1 - (1-\psi)^n$ for $0 < \psi < 1$. Hence, $P_n(\lambda) = e^{-\lambda} \frac{\lambda^n}{n!} \left(1 - (1-\psi)^n\right)$ for $n \in \mathbb{N}_1$ and $P_0(\lambda) = 1 - \sum_{n=1}^{\infty} P_n(\lambda) = e^{-\psi\lambda}$.

[53] Geromichalos (2012) analyzes minimum demand requirements in a different context. Minimum class size requirements are also common in the matching between students and schools.

This technology gives $\phi\left(\mu,\lambda\right) = 1 - e^{-\mu} - e^{-\psi\lambda} + e^{-\lambda\psi-\mu(1-\psi)}$. Hence, $\phi_\lambda = \psi e^{-\psi\lambda}\left(1 - e^{-\mu(1-\psi)}\right)$, which is strictly positive. However, the determinant of the Hessian of ϕ, evaluated in $\mu = 0$, equals $-\psi^2(1-\psi)^2 e^{-2\lambda\psi} < 0$, which means that ϕ is not concave. Hence, $\phi_\lambda \geq 0$ does not imply love for variety. □

CHAPTER 4

Entry efficiency and the meeting technology[54]

4.1 Introduction

This chapter analyzes the efficiency of vacancy creation in the presence of search frictions and information asymmetries. The presence of search frictions implies that it takes time for workers and firms to meet and thus vacancies are not filled immediately. Information asymmetry implies that workers or firms do not know how much the other party values a potential match. Specifically, I consider the following framework. In each period, each firm observes a private signal indicating the productivity and after observing this signal it needs to decide whether to enter or not. Upon entering, it opens one vacancy. Firms can fail to meet workers. Both unemployed and employed workers can receive a random number of job offers, including zero. The stochastic process governing job arrivals is exogenous and referred to as the meeting technology. Moreover, workers don't know their productivity at a potential match, but firms do.

The focus of this chapter is to study the impact of the meeting technology on the social efficiency of firm entry. To focus on the meeting technologies, I shut down other possible sources of inefficiency by making the following two assumptions: (i) unemployed workers and employed workers search

[54]This chapter is based on Cai (2015a)

equally efficient, and (ii) the wage mechanism is such that workers always choose the highest productivity firm and the firm with productivity equal to the workers' value of leisure has value zero. Examples include wage posting with commitment (see Burdett and Mortensen (1998)) or job auctions (see Julien et al. (2000)). If workers always move from less to more productive firms, the information friction does not cause any inefficiencies in multilateral meetings. Therefore, any possible inefficiency must come from externalities in the meeting technology. In general, different wage mechanisms affect firms' expected profits differently and firms' entry accordingly. By using the celebrated revenue equivalence result from the mechanism design literature and considering only efficient wage mechanisms, this chapter is able to analyze in both a static and a dynamic model how the meeting technology affects firm entry for a class of efficient wage mechanisms like wage posting or job auctions. If firms' productivity were public information, even with efficient wage mechanisms, the details of the mechanism, e.g., the protocols of wage bargaining, would also matter. Then the worker may still move to the most productive firm for a wide class of models (see, e.g., Pissarides (1994), Postel-Vinay and Robin (2002), and Gautier et al. (2010)), but depending on the specific wage mechanism, the efficiency implications vary from model to model.

Most papers that allow for many-to-one meetings focus only on the urn-ball meeting technology, i.e., the number of job offers that a worker receives in each period follows a Poisson distribution. Exceptions are, for example, Eeckhout and Kircher (2010b), Lester et al. (2015), and Cai et al. (2015). To explore the relation between search friction and firm entry, I allow the number of job offers a worker receives in each period to follow an arbitrary distribution. Cai et al. (2015) introduce an alternative representation of the meeting technology: a function which specifies the probability that a worker meets at least one firm from an arbitrary subgroup. This function makes the analysis of general meeting technologies as simple as the urn ball model. More importantly, it turns out that any externalities in the meeting process, positive or negative, can be naturally defined in terms of this function.

My main findings are that if an additional firm does not affect the meeting probabilities of existing firms, then the decentralized market is efficient; if an additional firm decreases (increases) the meeting probabilities of existing firms, then the decentralized equilibrium will have too much (little) firm entry. The externality in this market is unrelated to the fact that additional firms affect the *matching* probabilities of the existing firms, instead it operates through the *meeting* probabilities. The intuition is most easily understood in a one-period model where the wage is determined by a second-price auction among the firms that have contacted the worker. Consider a marginal entrant. If the firm's productivity is not the highest in an auction, then the direct contribution to surplus is zero and the firm's gain is also zero. Otherwise the firm's profit is the difference between the highest and the second highest productivity in the auction, which is also the firm's direct contribution to the aggregate surplus. Furthermore, if the marginal entrant does not affect the meeting probabilities of other workers and firms, then the private incentive for a firm to enter is perfectly aligned with the social planner's and thus in equilibrium firm entry is efficient. When a newly entering firm reduces the meeting probabilities of the incumbent firms, it imposes a negative externality and the marginal entrant's private gain is bigger than its contribution to surplus. Thus firm entry will be excessive. Of course, when there are externalities, it is not surprising that entry can be socially inefficient. The real contribution is that this chapter makes precise what a meeting externality is in a very general way. In addition, by considering general meeting technologies and general wage models, I contribute to the existing literature by clarifying and extending many of the existing results on entry efficiency. Finally, I introduce a new model of firm entry which allows for both ex ante and ex-post heterogeneity (see McAfee (1993), Wolinsky (1988), and Peters and Severinov (1997)) as special cases. I do this by by introducing signals about productivity that firms receive before entry. The case where the signals predict productivities perfectly corresponds to ex-ante heterogeneity, and the case where signals have no prediction power to ex-post heterogeneity. Intermediate cases arise between the two. For all cases I show that the equilibrium exists and is unique.

Diamond (1982) and Hosios (1990) are early papers analyzing the effects of meeting technologies on efficiency. Both papers consider wage bargaining with complete information, and don't allow for on-the-job search. Only in the cutting edge case where workers' bargaining power equals the elasticity of the aggregate matching function with respect to the number of unemployed workers, firm entry is efficient. Pissarides (1994) and Gautier et al. (2010) consider on-the-job search in the Diamond-Mortensen-Pissarides (DMP) framework, where workers and firms always bargain bilaterally and workers' outside option is the unemployment. Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006) consider an on-the-job search model where a worker can bargain with his current employer and a poacher simultaneously, and instead of unemployment, his outside option is the job with the second highest productivity. In the above mentioned on-the-job search models with bargaining, there are no general results relating entry efficiency and meeting technologies.

By assuming asymmetric information, this chapter distances itself from the above mentioned wage bargaining models. Butters (1977) considers a one-period, random search model where homogeneous workers and firms meet according to the urn-ball meeting technology. He shows that firm entry is efficient with wage posting. In his model, firms post and commit to a wage and workers do not (need to) know their values at potential matches. In a similar framework, Julien et al. (2000) show that the same efficiency result holds when workers are able to sell their service via an auction. Lester et al. (2015) and Cai et al. (2015) extend the above framework to allow buyer heterogeneity and they show that with invariant meeting technologies, firm entry is efficient in the ex-post and the ex-ante heterogeneity case, respectively. All the above results are special cases of the current chapter, and the equivalence can be easily interpreted by Proposition 15 below. In addition to the wage bargaining model mentioned above, Gautier et al. (2010) also consider the wage posting model of Burdett and Mortensen (1998) and show that if unemployed and employed workers have the same search intensity, then firm entry is efficient with the urn-ball meeting technology. This chapter shows, in the dynamic extension of the one-period model, that their result can be generalized to all invariant meeting technologies. It further shows how negative or positive externalities in the meeting process affect firm entry.

The chapter is organized as follows. Section 4.2 introduces the basic one-period model. Section 4.3 extends the model into a dynamic one. Section 5.4 concludes.

4.2 A One-Period Model

Agents. The economy consists of a measure n_f of firms and n_w of workers where I normalize the latter to one. Market tightness λ is defined as n_f/n_w. Both workers and firms are risk neutral. Workers are homogeneous and have value of leisure b. Firms differ by their productivity level x, with $b \le x \le c$. The nature of the heterogeneity of firms will be specified in details below.

Frictions. Each firm has one vacancy and tries to hire one worker, but the hiring process is frictional and governed by a meeting technology. Search is random, i.e., unlike in the previous chapter, firms do not ex ante observe the terms of trade. They learn it after meeting a worker. The meeting technology is constant-returns-to-scale and depends solely on market tightness λ. Each firm can at most meet one worker, but each worker can meet multiple firms. The probability of a worker meeting n firms is given by $P_n(\lambda)$, $n = 0, 1, \ldots$, where $P_n(\lambda)$ is assumed to be continuously differentiable. A natural requirement is that $P_0(0) = 1$, i.e., a worker can not meet any firm unless there exist some firms. The meeting technology treats different firms anonymously, i.e., independent of their productivity.[55] By an accounting identity, the probability of a firm to meet a worker who meets n firms in total is given by $Q_n(\lambda) = nP_n(\lambda)/\lambda$, $n = 1, 2, \ldots$. Therefore the probability that a firm fails to meet any worker is $Q_0(\lambda) = 1 - \sum_1^\infty Q_n(\lambda)$.

Two assumptions on meeting technologies. Cai et al. (2015) (see the previous chapter) defined an alternative representation of the meeting technology by introducing, for $0 \le \mu \le \lambda$,

$$\phi(\mu, \lambda) = 1 - \sum_0^\infty P_n(\lambda)(1 - \frac{\mu}{\lambda})^n. \tag{4.1}$$

Note that $\phi(\mu, \lambda)$ is always increasing and concave in μ. To understand the function ϕ, suppose we color an arbitrary fraction μ/λ of firms blue, and

[55] The same assumption is also adopted by Burdett and Judd (1983), Eeckhout and Kircher (2010b), Lester et al. (2015), and Cai et al. (2015).

suppose that the other firms are red, then $\phi(\mu,\lambda)$ is the probability that a worker meets at least one blue firm. For future use, note that

$$\phi_\mu(\mu,\lambda) = \sum_1^\infty Q_n(\lambda)(1-\frac{\mu}{\lambda})^{n-1}. \tag{4.2}$$

So $\phi_\mu(\mu,\lambda)$ is the probability that a firm successfully meets a worker who is in contact with no other blue firms. The function ϕ is an alternative representation of the meeting technology. As shown in Cai et al. (2015), the probability generating function $m(z,\lambda)$ which is defined by $\sum_{n=0}^\infty P_n z^n$ can be also written as $m(z,\lambda) = 1-\phi(\lambda(1-z),\lambda)$, and $P_n(\lambda)$ can be recovered from ϕ by noticing $P_n(\lambda) = \frac{1}{n!}\frac{\partial^n}{\partial z^n} m(z,\lambda)|_{z=0} = \frac{(-\lambda)^n}{n!}\frac{\partial^n}{\partial \mu^n}(1-\phi(\mu,\lambda))|_{\mu=\lambda}$.

Besides smoothness, we impose the following assumptions on the meeting technology, expressed in terms of ϕ.

Assumption 1. *For any z with $0 \le z < 1$, $\frac{d}{d\lambda}\phi(\lambda(1-z),\lambda) > 0$.*

Suppose that we divide firms by productivity into two groups: high and low productivity, with measure $\lambda(1-z)$ and λz respectively. Thus $\phi(\lambda(1-z),\lambda)$ is the probability of a worker to meet at least one high productivity firm. Assumption 1 implies that keeping fixed the ratio of high and low productivity firms, increasing the number of firms makes it easier for a worker to meet at least one high productivity firm. The same applies to meeting at least one low productivity firm.[56] In the special case with $z=0$, Assumption 1 states that $P_0(\lambda)$ is strictly decreasing, i.e., if the measure of firms increases, it becomes easier for a worker to meet at least one firm.

Assumption 2. *For any z with $0 \le z < 1$, $\frac{d}{d\lambda}\phi_\mu(\lambda(1-z),\lambda) < 0$.*

If we divide firms into high and low productivity ones as before, by equation (4.2) $\phi_\mu(\lambda(1-z),\lambda)$ is the probability that a firm successfully meets a worker whose other possible contacts are all low productivity firms. Assumption 2 implies that as there are more firms available, it becomes harder for a firm to meet a worker who has not been approached by a high productivity firm. Intuitively, it means that when market tightness is high, it is less likely that a high productivity firm improves the surplus.[57] In the special case with $z=0$, by equation (4.2) Assumption 2 states that $Q_1(\lambda)$ is strictly decreasing.

[56] Assumption 1 is also adopted by Eeckhout and Kircher (2010b) and Lester et al. (2015), and it can also be stated in terms of P_n, i.e., $\sum_0^\infty P_n'(\lambda)z^n > 0$.

[57] Assumption 2 can also be stated in terms of Q_n, i.e., $\sum_1^\infty Q_n'(\lambda)z^n < 0$.

Examples of meeting technologies. Below we list some examples of meeting technologies that satisfy Assumptions 1 and 2. Details of verification can be found in the appendix.

1. (Urn-ball.) The urn-ball meeting technology is given by $P_n(\lambda) = e^{-\lambda}\lambda^n/n!$ for $n = 0, 1, \ldots$. $\phi(\mu, \lambda) = 1 - e^{-\mu}$ and $\phi_\lambda = 0$.
2. (Bilateral.) A bilateral meeting technology is one with $P_n(\lambda) = 0$ for $n \geq 2$, and $P_0(\lambda)$ strictly decreasing and concave. $\phi(\mu, \lambda) = P_1(\lambda)\mu/\lambda$ and $\phi_\lambda < 0$.
3. (Cobb-Douglas.) The Cobb-Douglass meeting technology is commonly used as an aggregate matching function.[58] $P_n(\lambda) = e^{-\lambda^\alpha}(\lambda^\alpha)^n/n!$ for $n = 0, 1, \ldots$ with $0 \leq \alpha \leq 1$. Therefore, $\phi(\mu, \lambda) = 1 - e^{-\frac{\mu}{\lambda^{1-\alpha}}}$, and $\phi_\lambda < 0$.
4. (Mixture between urn-ball and pairwise urn-ball.) Suppose that with probabilities ζ and $1 - \zeta$, $0 < \zeta < 1$, an agent (a workers or a firm) will be allocated to submarkets A and B, respectively. In submarket A, workers and firms meet according to the urn-ball model. In submarket B, workers and firms meet according to the pairwise urn-ball model introduced by Lester et al. (2015). That is, in submarket B, firms first need to form pairs, and then as a pair, two firms will jointly meet a worker randomly. Formally, in submarket A, $P_n(\lambda \mid A) = e^{-\lambda}\frac{\lambda^n}{n!}$ for $n = 0, 1, 2, \ldots$, and in submarket B, $P_n(\lambda \mid B) = e^{-\lambda/2}\frac{(\lambda/2)^n}{(n/2)!}$ for $n = 0, 2, 4, \ldots$ and $P_n(\lambda \mid B) = 0$ for $n = 1, 3, 5, \ldots$. Therefore, for the aggregate matching market, $\phi(\mu, \lambda) = 1 - \zeta e^{-\mu} - (1 - \zeta)e^{-\mu(1 - \frac{\mu}{2\lambda})}$ and $\phi_\lambda > 0$.

Invariance. Lester et al. (2015) introduced an important generalization of the urn-ball meeting technology: invariant meeting technologies. Cai et al. (2015) show that invariant meeting technologies can be defined by $\phi_\lambda(\mu, \lambda) = 0$. To understand this condition, suppose that, as before, we color an arbitrarily fraction of μ/λ firms blue and the rest red. Invariant meeting technologies imply that the probability for a worker to meet at least one blue firm depends only on the measure of blue firms μ, and does not depend on the measure of red firms $\lambda - \mu$. Therefore in terms of meeting probabilities, firms do not impose any externalities upon each other when the meeting technology is invariant .

[58] See Petrongolo and Pissarides (2001) for a review of the aggregate matching function.

Wage determination. I use the language of mechanism design to make the wage model general. A wage model is a direct, anonymous mechanism that assigns a worker to a firm which has met him and specifies the relevant payoffs as a function of the total number of firms that the worker is in contact with and their reported productivities. Specifically, if firms $1,\ldots,n$ report $x=(x_1,\ldots,x_n)$, then the mechanism will allocate the worker to firm i with probability $\theta_n(x_i,x_{-i})$, and the expected payoff to firm i is $\eta_n(x_i,x_{-i})$. Note that because of the anonymity assumption, the wage mechanism allocates the worker and determines the relevant payoffs based on the reported productivities only, and the identities of firms do not matter. In this chapter, I only consider efficient mechanisms, i.e., a worker will always choose the most productive firm.[59]

Examples of efficient wage mechanisms under random search are: (i) wage posting and (ii) job auctions. In the wage posting model, firms commit to the wage they are willing to pay. When a worker meets a firm, he observes the wage that the firm offers and commits to. In the appendix I show that higher productivity firms will always post higher wages. Therefore, wage posting is an efficient wage mechanism. In the auction model, when a worker meets multiple firms, the worker will hold a second-price auction with reserve price b, and firms will bid the worker's service. As a result, the second-highest bid will be the worker's wage. Therefore, an auction is also an efficient wage mechanism.[60]

Free entry. There exists a large measure Λ of firms considering whether to enter the market or not. To enter the market, firms must incur a cost K. Before entry, each firm observes a private signal s, $b \le s \le c$, with distribution $L(s)$. For a firm with signal s, after entering the market, the firm will draw a productivity x from a cumulative probability distribution (cdf) $H(x \mid s)$ with $b \le x \le c$. A better signal means that the firm is more likely to draw a better productivity. As we will see later, in equilibrium there will exist a threshold signal: firms will

[59] In the case of multiple firms having the highest productivity, a worker will randomize with equal probabilities.

[60] See Butters (1977) and Julien et al. (2000) for a wage posting and an auction model, respectively.

enter the market only if their signals are above this threshold. To show that the equilibrium is unique, I impose the following structure on $H(x \mid s)$.

Assumption 3. *There exists a nondecreasing function $\overline{x}(s)$ and a cumulative distribution function $\widetilde{H}(z)$ such that $H(x \mid s)$ is the cumulative distribution function corresponding to $x = \min(\overline{x}(s), z)$, where z is a draw from $\widetilde{H}(z)$.*

Before entry, the signal s that a firm observes determines the highest possible productivity the firm could have, $\overline{x}(s)$. After entry, a firm will first draw a productivity z from the distribution $\widetilde{H}(z)$. If the draw z is smaller than $\overline{x}(s)$, then z will be the firm's realized productivity. If the draw z is larger than $\overline{x}(s)$, then $\overline{x}(s)$ will be the firm's realized productivity. Therefore, one consequence of Assumption 3 is that if $x < \overline{x}(s)$, then

$$1 - H(x \mid s) = 1 - \widetilde{H}(x), \tag{4.3}$$

where the right hand side is independent of s.

The above specification for firm entry generalizes the two commonly used entry models in the literature:[61] (i) ex-ante heterogeneity and (ii) ex-post heterogeneity. Under (i), all firms are ex-ante heterogeneous. There is a large number Λ of firms whose productivity follows a given distribution $F(x)$. Each firm already knows its productivity and must decide whether to pay a cost K and enter the market. Under (ii), all firms are ex-ante homogeneous. There is a large number of potential entrants. Firms have to pay cost K to enter the market, and after paying the cost, the productivity will be drawn from an exogenous distribution $F(x)$. It is easy to see that the ex-ante heterogeneity case corresponds to $L(s) = F(s)$, $\overline{x}(s) = s$ for $b \leq s \leq c$, and $\widetilde{H}(x) = 0$ for $b \leq x < c$ and $\widetilde{H}(c) = 1$, and the ex-post heterogeneity case corresponds to $\overline{x}(s) = c$ and $\widetilde{H}(x) = F(x)$.

Equivalence of wage mechanisms

In the following I assume that the measure and the productivity distribution of firms in the market are given, i.e., the entry decisions have been made.

[61] see, e.g., Wolinsky (1988), McAfee (1993), Peters and Severinov (1997), and Albrecht et al. (2014).

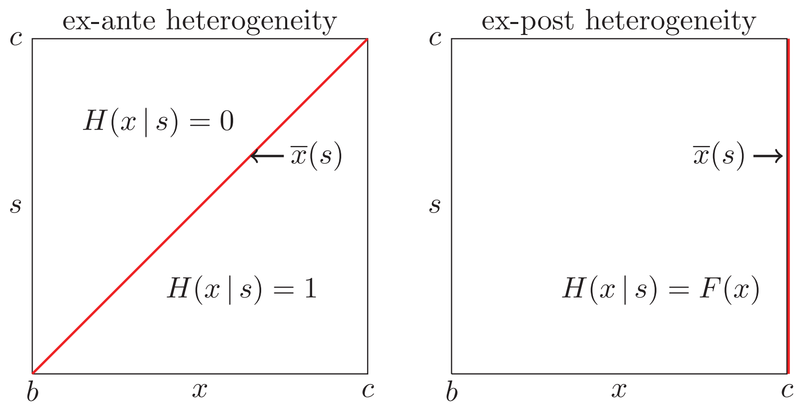

Figure 9: Ex-ante and ex-post heterogeneity as special cases

But it should be noted that both variables are dependent on the threshold of signals s^*. By assuming that the wage mechanism is efficient and firms with productivity b have value 0, I will first derive in equations (4.4), (4.5), and (4.7) below the total output in the market, the marginal contribution to surplus of a firm with productivity x, and the expected payoff for a firm with productivity x respectively. Detailed derivations of the above equations can be found in Cai et al. (2015). See also the appendix.

Total output can be divided into two parts: (i) workers' leisure value b; (ii) the productivity gain when workers are employed. A n-to-1 meeting is a scenario where a worker meets n firms. Because the wage mechanism is assumed to be efficient, the distribution of productivity gains, $z-b$, of a worker conditional on a n-to-1 meeting is $F^n(z)$. Therefore, total output is given by

$$S = b + \sum_{n=1}^{\infty} P_n(\lambda) \int_b^c (z-b)\, dF^n(z) = b + \int_b^c \phi(\lambda(1-F(z)), \lambda)\, dz, \tag{4.4}$$

where $\phi(\lambda(1-F(z)), \lambda)$ is the probability that a worker meets at least one firm with productivity higher than z. Thus the total output in addition to the leisure value b can be expressed by integration of ϕ.

Next, consider the marginal contribution to the total output by a firm of productivity x. A firm can affect total output in two ways: (i) indirectly or (ii) directly. Under (i), a marginal firm will affect meeting probabilities (function ϕ) through market tightness, and thus total output. Under (ii), a firm contributes directly to the total output if it has the highest productivity in an n-to-1 meeting. Thus the marginal contribution to the total output by a firm of productivity x is

$$T(x) = \int_b^c \phi_\lambda(\lambda(1-F(z)),\lambda)dz + \int_b^x \phi_\mu(\lambda(1-F(z)),\lambda)dz. \tag{4.5}$$

where the first term corresponds to the indirect effects and the second term to the direct effects.

Now we move to the calculation of the expected value of firms. Consider a firm with productivity x who is in an n-to-1 meeting. From firm x's perspective, the probability of this event is $Q_n(\lambda)$, and conditional on this, the winning probability for the firm is $F^{n-1}(x)$ since we consider only efficient mechanisms. By Myerson (1981), the incentive compatibility constraint implies that the expected payoff for a firm of productivity x in an n-to-1 meeting is determined by the winning probabilities (payoff equivalence) and is given by

$$J_n(x) = J_n(b) + \int_b^x F^{n-1}(z)dz. \tag{4.6}$$

Therefore, the expected payoff for a firm is

$$J(x) = \sum_{n=1}^{\infty} Q_n(\lambda)J_n(x) = \int_b^x \phi_\mu(\lambda(1-F(z)),\lambda)dz. \tag{4.7}$$

Note that $\phi_\mu(\lambda(1-z),\lambda)$ is the probability that a firm meets a worker whose other possible contacts all have productivity lower than z, and therefore it is the probability of a firm with productivity z winning a worker.

The expected value of a worker can be calculated as the residual between the total output and the aggregate firm value. Therefore, the expected worker value is

$$V = S - \lambda \int_b^c J(x)dF(x). \tag{4.8}$$

As noted before, under random search both wage posting and job auctions are efficient wage mechanisms. Furthermore, in both mechanisms, firms with productivity b always have zero value, $J(b) = 0$, since b is also the workers' leisure value. Therefore, wage posting and job auctions are equivalent in terms of expected values for workers and firms. This result is a generalization of Kultti (1999) and Eeckhout and Kircher (2010b), who, in a model with homogeneous workers and firms, showed the equivalence result for the urn-ball meeting technology and general meeting technologies, respectively .

Proposition 15. *The expected payoffs for workers and firms of each type are exactly the same for all efficient wage mechanisms with $J(b) = 0$.*

Proof. See the previous discussion. □

Entry and efficiency

Since invariant meeting technologies play an important role in the analysis, I will first show that they satisfy both Assumption 1 and 2. First we need the following simple lemma. The proof can be found in the appendix.

Lemma 16. *With invariant meeting technologies, $P_0(\lambda) + P_1(\lambda) < 1$ for any $\lambda > 0$, and $\phi(\mu,\lambda)$ is strictly concave in μ.*

Since invariant meeting technologies are defined by $\phi_\lambda(\mu,\lambda) = 0$, we have $d\phi(\lambda(1-z),\lambda)/d\lambda = (1-z)\phi_\mu(\lambda(1-z),\lambda)$ and $\frac{d}{d\lambda}\phi_\mu(\lambda(1-z),\lambda) = (1-z)\phi_{\mu\mu}(\mu,\lambda)$. Because $\phi(\mu,\lambda)$ is increasing and strictly concave in μ, $\phi_\mu(\lambda(1-z),\lambda) > 0$ and $\phi_{\mu\mu}(\lambda(1-z),\lambda) < 0$. Therefore, Assumption 1 and 2 are easily satisfied.

In Section 4.2 market tightness λ and the productivity distribution of firms in the market $F(x)$ are treated as given. Here we will show how both market tightness and the productivity distribution of firms after entry depend on the threshold of signals, s^*. To emphasize their dependence on s^*, with a slight abuse of notation I append s^* as an additional argument to functions introduced in Section 4.2, e.g., $F(x)$ becomes $F(x,s^*)$.

If only firms with signals higher than s^* enter, then market tightness is simply given by

$$\lambda(s^*) = \Lambda\left(1 - L(s^*)\right). \tag{4.9}$$

The distribution of firm productivity F is given by the following equation,

$$\lambda(s^*)\left(1 - F(x,s^*)\right) = \Lambda\int_{s^*}^{c} 1 - H(x\,|\,s)\,dL(s), \tag{4.10}$$

where the left hand side is the total measure of firms with realized productivity above x, and $1 - H(x\,|\,s)$ at the right hand side is the probability of an entrant with signal s having a realized productivity above x. Therefore, by equations (4.9) and (4.10) and the chain rule of differentiation, for $b \le x \le c$ we have

$$\frac{d}{ds^*}\phi(\lambda(s^*)(1-F(x,s^*),\lambda(s^*)) = -\Lambda l(s^*)\left((1-H(x\,|\,s^*))\phi_\mu + \phi_\lambda\right). \tag{4.11}$$

where $l(s)$ is the probability density function of $L(s)$. The above equation gives the response of the workers' probability to meet at least one firm with productivity higher than x with respect to the entry threshold s^*.

For a firm with signal s, since after entry the firm's productivity distribution is $H(x\,|\,s)$, before entry the expected contribution to total output $\widetilde{T}(s)$ is given by

$$\begin{aligned}\widetilde{T}(s) &= \int_b^c T(x,s^*)\,dH(x\,|\,s) = T(b,s^*) + \int_b^c (1-H(x\,|\,s))\,dT(x,s^*)\\ &= \int_b^c \phi_\lambda dx + \int_b^c (1-H(x\,|\,s))\,\phi_\mu dx,\end{aligned} \tag{4.12}$$

where $T(x,s^*)$ is the marginal contribution to surplus of a firm with productivity x and is given by equation (4.5), in which λ and $F(x)$ are given by equations (4.9) and (4.10) respectively, and in the second equality of the first line we used integration by parts, and the equality in the second line is because $dT(x,s^*) = \phi_\mu(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*))dx$ by equation (4.5). Note that in the second line the argument of $\phi(\Lambda\int_{s^*}^c 1-H(x\,|\,s)dL(s),\Lambda(1-L(s^*)))$ is suppressed to save space. In the following, I will suppress the arguments of function ϕ whenever it doesn't cause confusion. The interpretation of equation (4.12) is similar to that of equation (4.5), except that now the firm's productivity is determined by the distribution $H(x\,|\,s)$.

Similarly, for any decentralized equilibrium with efficient wage mechanisms, the expected value of a firm with signal s before entry is given by,

$$\begin{aligned}\widetilde{J}(s) &= \int_b^c J(x,s^*)\,dH(x\,|\,s) = \int_b^c (1-H(x\,|\,s))\,dJ(x,s^*)\\ &= \int_b^c (1-H(x\,|\,s))\,\phi_\mu dx,\end{aligned} \tag{4.13}$$

where $J(x,s^*)$ is the expected value of a firm with productivity x which is given by equation (4.7), with the understanding that now market tightness and the productivity distribution of firms are both determined by the signal threshold

s^*. Here again in the first line we used integration by parts together with the fact $J(b,s^*)=0$ because of our assumption on the wage mechanism, and the equality in the second line is because $dJ(x,s^*)=\phi_\mu(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*))dx$ by equation (4.7).

To establish uniqueness of the decentralized equilibrium, we need the following lemmas.

Lemma 17. *For a given x with $b \le x < \overline{x}(s^*)$, $\frac{d}{ds^*}\phi(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*)) < 0$*

.

Proof. If $x < \overline{x}(s^*)$, then $x < \overline{x}(s)$ for any $s > s^*$ because $\overline{x}(s)$ is nondecreasing by our assumption on the entry (Assumption 3). By equation (4.3), for any $s \ge s^*$ and $x < \overline{x}(s^*)$, $1-H(x\,|\,s) = 1-\widetilde{H}(x)$. Therefore, by equation (4.10) we have

$$1-F(x,s^*) = 1-\widetilde{H}(x), \tag{4.14}$$

which is independent of s^*. Therefore,

$$\frac{d}{ds^*}\phi(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*)) = \frac{d\lambda(s^*)}{ds^*}\frac{d}{d\lambda(s^*)}\phi(\lambda(s^*)(1-\widetilde{H}(x)),\lambda(s^*))$$

With a higher entry threshold s^*, market tightness $\lambda(s^*)$ will decrease because of fewer entry. So the first term at the right hand of the above equation is negative. By Assumption 1, the second term at the right hand side is positive. The result then follows. □

Lemma 17 implies that with a higher signal threshold of entry, it become more difficult for a worker to meet at least one firm with productivity higher than x, if $x < \overline{x}(s^*)$. With ex-post heterogeneity, $\overline{x}(s^*)$ is always c, the highest possible productivity, and Lemma 17 is equivalent to Assumption 1.

Lemma 18. *For a given x with $b \le x < \overline{x}(s^*)$, $\frac{d}{ds^*}\phi_\mu(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*)) >$ 0.*

Proof. The proof is similar to that of Lemma 17. Since $x < \overline{x}(s^*)$, by equation (4.14) we have $1 - F(x, s^*) = 1 - \widetilde{H}(x)$. Therefore,

$$\frac{d}{ds^*}\phi_\mu(\lambda(s^*)(1-F(x,s^*),\lambda(s^*)) = \frac{d\lambda(s^*)}{ds^*}\frac{d}{d\lambda(s^*)}\phi_\mu(\lambda(s^*)(1-\widetilde{H}(x)),\lambda(s^*))$$

Same as in the proof of Lemma 17, the first term at the right hand side is negative. By Assumption 2, the second term at the right hand is negative. The result then follows. □

The relation between Lemma 18 and Assumption 2 is similar to that between Lemma 17 and Assumption 1. Lemma 18 implies that with a higher signal threshold of entry, it becomes easier for a firm to meet a worker who has not been contacted by firms with productivity higher than x, if $x < \overline{x}(s^*)$. Thus, the winning probability of a high productivity firm is larger with a higher signal threshold.

Lemma 19. *$\widetilde{J}(s^*)$ is an increasing function of s^*.*

Proof. First by equation (4.13), $\widetilde{J}(s^*) = \int_b^c (1 - H(x\,|\,s^*))\,\phi_\mu(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*))dx$. By Assumption 3, $1 - H(x\,|\,s^*) = 0$ if $x \geq \overline{x}(s^*)$, and by equations (4.3) and (4.14), $1 - H(x\,|\,s^*) = 1 - F(x, s^*) = 1 - \widetilde{H}(x)$ if $x < \overline{x}(s^*)$. Therefore,

$$\widetilde{J}(s^*) = \int_b^{\overline{x}(s^*)} (1-\widetilde{H}(x))\phi_\mu(\lambda(s^*)(1-\widetilde{H}(x)),\lambda(s^*))dx,$$

where we used the fact that the integrand is 0 for $x \geq \overline{x}(s^*)$. Because of Lemma 18, $\phi_\mu(\lambda(s^*)(1-\widetilde{H}(x)),\lambda(s^*))$ is an increasing function of s^*. Also the upper bound of the integration range $\overline{x}(s^*)$ is nondecreasing in s^*. Therefore, $\widetilde{J}(s^*)$ is increasing in s^*. □

Lemma 19 implies that with a higher signal threshold of entry, the value of the marginal entrant increases. The intuition is most clear for the two extreme cases: ex-post and ex-ante heterogeneity. With ex-post heterogeneity, this means that with less competition on the firm side, firm value increases. With ex-ante heterogeneity, the lowest productivity firm in the market wins a worker if it is the only contact the worker has. A higher threshold implies two things: (i) The probability of a firm being the only contact that a worker has increases,[62] (ii) conditional on winning, the value is larger because of the higher productivity. Therefore, $J(s^*)$ is increasing in s^*. Lemma 19 is the basis for uniqueness of the equilibrium

Both the planner's problem and the decentralized equilibrium can be characterized in a very simple way. The planner will require that for the firm at the threshold s_p^*, the marginal contribution to the total output $\widetilde{T}(s_p^*)$ is equal to the entry cost K. In the decentralized equilibrium, the marginal firm with signal s_e^* is such that the expected value of entry $\widetilde{J}(s_e^*)$ is equal to the entry cost K. Therefore, the condition for the optimal s_p^* is

$$K = \widetilde{T}(s_p^*) = \int_b^c \phi_\lambda dx + \int_b^c \left(1 - H(x \mid s_p^*)\right) \phi_\mu dx \tag{4.15}$$

The equilibrium threshold of signals, s_e^*, is given by

$$K = \widetilde{J}(s_e^*) = \int_b^c \left(1 - H(x \mid s_e^*)\right) \phi_\mu dx \tag{4.16}$$

Proposition 20. *There is a unique equilibrium in the decentralized market.*

1. *If $\phi_\lambda = 0$, i.e., with invariant meeting technologies, firm entry is efficient.*
2. *If $\phi_\lambda < 0$ (> 0), in equilibrium there will be too much (little) firm entry.*

Proof. The uniqueness of the equilibrium follows directly from equation (4.16) and Lemma 19.

With invariant meeting technologies, $\phi_\lambda(\mu, \lambda) = 0$. Comparing equations (4.5) and (4.7) gives $J(x, s^*) = T(x, s^*)$. Therefore, $\widetilde{J}(s) = \widetilde{T}(s)$ and $s_e^* = s_p^*$.

If $\phi_\lambda(\mu, \lambda) < 0$, then by equations (4.5) and (4.7), $T(x, s^*) < J(x, s^*)$. That is, for a firm of productivity x, the expected contribution to the total output is smaller than the expected value for the firm. Since $T(x, s^*) < J(x, s^*)$, we also have $\widetilde{T}(s) < \widetilde{J}(s)$. Since $K = \widetilde{T}(s_p^*) < \widetilde{J}(s_p^*)$ and $K = \widetilde{J}(s_e^*)$, we have $\widetilde{J}(s_e^*) < \widetilde{J}(s_p^*)$ and by Lemma 19, $s_p^* > s_e^*$, which implies in equilibrium there is excessive firm entry. The same argument applies to the case $\phi_\lambda(\mu, \lambda) > 0$. □

The intuition for the first part of Proposition 20 is as follows. With an invariant meeting technology, firms do not impose meeting externalities on each other. Suppose that the wage mechanism is a second-price auction with

[62] Formally, $Q_1(\lambda)$ is strictly decreasing because of Assumption 2.

a reserve price b, and the highest and the second highest productivity is x_n and x_{n-1}, then the social surplus generated by the highest productivity firm x_n is $x_n - x_{n-1}$, since if the firm did not enter, the worker would have gone to the second highest productivity firm. This is exactly the firm's payoff in the second-price auction. Therefore, firm entry is efficient. For other wage mechanisms, because of the equivalence result in Proposition 15, as long as the wage mechanism is efficient and $J(b) = 0$, all wage mechanisms are equivalent in terms of expected payoffs and firm entry is efficient. Therefore, despite the fact that wage posting and job auctions are different wage models and have different implications for the wage distribution, they generate exactly the same expected values for workers and firms, and thus the same entrants.[63]

To the best of my knowledge, the second part of Proposition 20 is new in the literature. If invariance is violated, depending on the nature of the externality, there might be too much or too little firm entry. The result is intuitive: if firms impose negative effects on each other in meeting workers, then their social contribution is smaller than their private values and thus there will be too much entry. The condition $\phi_\lambda < 0$ (or > 0) makes the meaning of negative (or positive) externality formal. Unlike in the directed search case where externalities can be priced and firm entry is always efficient (see Cai et al. (2015)), under random search externality can cause too much or too little entry.

4.3 A Dynamic Model

Here I will extend the basic one-period model to a dynamic one, and show that the conclusions from the one-period model continue to hold. I maintain

[63] Special cases of the first part of Proposition 20 appear in different papers in the literature. The closest are Lester et al. (2015) and Cai et al. (2015), which show that with invariant meeting technologies, firm entry is efficient in the ex-post and the ex-ante heterogeneity case, respectively. Here I allow for a more general model of firm entry.

With homogeneous firms and workers and the urn-ball meeting technology, Butters (1977) shows that firm entry is efficient with wage posting. Butters wrote:"I don not know of a simple way to explain why the market is efficient..." See p478 of Butters (1977). In view of Proposition 15 and 20, the efficiency result is trivial. Again with homogeneous firms and workers and the urn-ball meeting technology, Julien et al. (2000) show that firm entry is efficient with the auction model. Albrecht et al. (2014) show that with the urn-ball meeting technology and heterogeneous firms, firm entry is efficient in the auction model for both the ex-post and the ex-ante heterogeneity case.

Assumption 1, 2, and 3. In the following I will sketch the dynamic model with emphasis on the new features introduced by the dynamic extension.

Time is discrete and indexed by $t = 0, 1, \ldots$. The discount rate for both workers and firms is $1/(1+\rho)$. Following Gautier et al. (2010), I also assume that the population growth rate equals the discount rate, i.e., the golden growth assumption. As noted by Gautier et al., this is similar to the assumption of no discouting commonly used in the search literature. See, e.g., Hosios (1990) and Burdett and Mortensen (1998). So in each period there are ρ newborns. There is no death of workers or firms.

Each firm consists of a single job. Opening a vacancy costs K each period. As in the one-period model, in each period there is a large measure Λ of potential entrants. At the beginning of each period, each potential entrant receives a private signal s from a distribution $L(s)$, and the job's productivity x is distributed according to $H(x \mid s)$. After observing the signal, each firm needs to decide whether to stay in the market for the period or to leave. The productivity x will stay constant throughout the duration of the job. If the vacancy is not filled this period, then next period the firm will receive another private signal and decide again whether to enter the market or not. For simplicity, I assume signals of different periods are independent, identically distributed.

In each period, both unemployed and employed workers receive job offers according to the meeting technology $P_n(\lambda)$, where λ is market tightness and $n = 0, 1, \ldots$.[64] As before I will only consider efficient wage mechanisms. Together with the assumption that unemployed and employed workers meet firms equally efficient, this implies that in each period a worker will always work for the firm (from the set of firms that he met) with the highest productivity.

The rest of the model will be specified by following a newborn worker. Suppose a worker is born at the beginning of period 0. He enjoys b for period 0, and at the end of the period he is contacted by a random set N_0 of firms. If

[64] It is a strong assumption that unemployed and employed workers meet firms in the same way. However, in this chapter I am mainly interested in how the externality caused by firms in meeting workers affects the social efficiency. If unemployed workers search more efficiently, then there will be a productivity threshold below which unemployed workers will not accept jobs. The threshold will in general be inefficient. This will complicate the analysis without contributing much to the main focus of the chapter. The same assumption with the urn-ball meeting technology is adopted by Mortensen (2003) and in a continuous-time framework by Gautier et al. (2010).

N_0 is nonempty, then each firm in the set N_0 will report a productivity level. The wage mechanism is anonymous, direct, and incentive compatible. It will determine the allocation of the worker's service of period 1, along with relevant payoffs, based on the reports of firms in N_0. If hired by some firm at the end of period 0, the worker will work for the firm in period 1. At the end of period 1, the worker will meet a new set N_1 of firms. These firms will also report their productivity. The wage mechanism will determine the allocation and payoffs in period 2 based on the reports of firms in sets N_0 and N_1.[65] The same process continues ad infinitum. Since I consider an economy of its balanced growth path (see Gautier et al. (2010)) where all new workers start as unemployed, I can leave out the job destruction shock, for adding an exogenous job destruction shock will not change anything.

As in the one-period model, I consider two common wage mechanisms that are efficient: wage posting and job auctions. Under wage posting, in each period after entry and the realization of the productivity, a vacancy will post and commit to a wage. If the vacancy successfully hires a worker, then the wage will remain constant until the worker leaves the firm. If the firm fails to hire, then at the beginning of the next period, after learning the private signal, the firm will decide whether to enter the market or not, and the game repeats the last period. In the auction model, at the end of each period, the worker will hold a second price auction for his next period's labor service. Suppose by the end of period t the highest and the second highest productivity the worker has encountered since he was born are x_1 and x_2 respectively, then he will work for firm x_1 with wage x_2 for period $t+1$. Note that neither x_1 nor x_2 need to be new contacts the work meet in period t. The auction model here is different from that of Postel-Vinay and Robin (2002), where the worker's wage will be less than x_2. The latter model assumed full information and firms to be able to make take-it-or-leave-it offers.

Since we only consider efficient mechanisms, job-to-job transitions are always efficient. Suppose that the entry threshold of signals is s^*, then

[65] The wage mechanism determines the allocation and payoffs based on both N_0 and N_1 does not necessarily mean that the worker or the wage mechanism can recall firms in N_0. In the model, there is no need for recall because we only consider efficient wage mechanisms and the firm with highest productivity that a worker has met is always his or her current employer or a new contact in the present period.

again market tightness $\lambda(s^*)$ and the distribution of each period's realized productivity of vacancies are given by equations (4.9) and (4.10), respectively. In the following we use the convention that unemployed workers are employed at jobs with productivity b. Denote the steady state distribution of productivity by $G(x,s^*)$, then the balance of labor market flows implies

$$\rho - G(x,s^*)\phi(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*)) = \rho G(x,s^*)$$

where the first term of the left hand side represents the new cohort, the second term captures the workers who move to a job with a productivity greater than z from a job lower than z, and the right hand side captures the increase in the number of workers with productivity lower than z. The above equation implies,

$$G(x,s^*) = \frac{\rho}{\rho+\phi} \tag{4.17}$$

where we have suppressed the arguments of function $\phi(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*))$. Since $\phi(0,\lambda^*(s)) = 0$, evaluating $x = c$ in equation (4.17) naturally gives $G(c,s^*) = 1$. Note that the unemployment rate is given by $G(b,s^*) = \rho/(\rho+\phi(\lambda^*,\lambda^*))$, where $\phi(\lambda^*,\lambda^*)$ is the probability of a worker to meet at least one firm in a period.

Planner's problem

Because of the golden-growth assumption, the social planner will maximize the flow value of net output by choosing an optimal entry threshold of signals s^*. Since the steady state productivity distribution of workers is $G(x,s^*)$, where unemployed workers are said to have productivity b, the flow value of total output is

$$S = \int_b^c x dG(x,s^*) = b + \int_b^c 1 - G(x,s^*)dx \tag{4.18}$$

where $G(x,s^*)$ is given by equation (4.17).

As mentioned above, the social planner's objective is to maximize the flow value of net output,

$$\Omega = S - \Lambda L(s^*).$$

Thus the planner's problem is solved by the following first-order condtion

$$\begin{aligned} K &= \frac{1}{\Lambda l(s^*)} \frac{\partial S}{\partial s^*} = \frac{1}{\Lambda l(s^*)} \int_b^c \frac{d\,(1 - G(x, s^*))}{d\phi} \frac{d\phi}{ds^*} dx \\ &= \int_b^c \frac{\rho}{(\rho + \phi)^2} \big((1 - H(x \,|\, s^*))\phi_\mu + \phi_\lambda\big)\, dx \end{aligned} \tag{4.19}$$

where $l(s)$ is the density function of $L(s)$, $G(x, s^*)$ is given by equation (4.17), and in the second line we used equation (4.11) to substitute out $d\phi/ds^*$, where the arguments of $\phi(\lambda(s^*)(1 - F(x, s^*)), \lambda(s^*))$ are suppressed. Compared to the optimality condition of the static model, equation (4.15), the optimality condition of the dynamic model, equation (4.19), differs by a factor $\rho/(\rho + \phi)^2$, which denotes the effect of on-the-job search on channeling ϕ, the probability of a worker to meet at least one firm with productivity higher than x in one period, into the steady state productivity distribution $G(x, s^*)$.

Decentralized equilibrium

To solve the firms' entry problem, I will first show how to extend the revenue equivalence results of Proposition 15 to the dynamic model. Since each firm reports their productivity to the wage mechanism only once, in the following I will show that the incentive-compatibility constraint for a firm is the same as the static one. Suppose a firm reports a productivity z in period 0, then in period 1 the probability that the firm successfully hires a worker is

$$R_1(z) = G(z, s^*)\phi_\mu(\lambda(s^*)(1 - F(z, s^*)), \lambda(s^*)), \tag{4.20}$$

where $\phi_\mu(\lambda(s^*)(1 - F(z, s^*)), \lambda(s^*))$ is the probability that the firm successfully meets a worker whose other job offers received in period 0 have productivity lower than z, and $G(z, s^*)$ is the probability that the contacted worker, if any, is currently employed at a job with productivity lower than z (unemployed workers are said to be employed at productivity b). For $t \geq 2$, $R_t(z) = R_{t-1}(z)(1 - \phi(\lambda(s^*)(1 - F(z, s^*)), \lambda(s^*)))$, where $R_{t-1}(z)$ is the probability that the worker is employed by the firm in period $t-1$ and $(1-\phi)$ is the probability that in period $t-1$, all job offers the worker received have productivity lower than z. Therefore, for $t \geq 1$,

$$R_t(z) = \left(1-\phi\right)^{t-1} R_1(z). \tag{4.21}$$

The present value of the output for a firm with productivity x reporting z is thus $xR(z)$ with

$$R(z) = \sum_{t=1}^{\infty} \frac{1}{(1+\rho)^t} R_t(z) = \frac{G(z,s^*)\phi_\mu}{\rho+\phi} = \frac{\rho}{(\rho+\phi)^2}\phi_\mu. \tag{4.22}$$

where $G(z,s^*)$ is given by equation (4.17). Therefore, the present value of a firm with productivity x reporting z can be written as $xR(z) - W(z)$, where $W(z)$ is the present value of the wage cost of reporting z. Because of the incentive-compatibility constraint, we only have to consider the function $R(z)$, which is similar to the winning probability ϕ_μ (see equations (4.2) and (4.7)) in the static model. Therefore, we are back in the static mechanism design problem. Note that $R(z)$ and ϕ_μ differ again by a factor $\rho/(\rho+\phi)^2$, which is caused by on-the-job search in the dynamic model. Similar to equation (4.7), the present value of a vacancy with productivity x is,

$$J(x,s^*) = \int_b^x R(z)dz = \int_b^x \frac{G(z,s^*)\phi_\mu}{\rho+\phi} dz. \tag{4.23}$$

where we used the assumption that $J(b,s^*) = 0$. Therefore, the equivalence result from the one-period model continues to hold for the dynamic setup, and we have a dynamic generalization of Proposition 15.

Proposition 21. *Suppose market tightness and the productivity distribution of vacancies are given, then the expected payoffs for workers and firms of each type are exactly the same for all efficient wage mechanisms with* $J(b,s^*) = 0$.

Proof. See the previous discussion. □

The threshold of signals in the decentralized equilibrium is determined by the following zero profit condition,

$$
\begin{aligned}
K &= \int_b^c J(x,s^*)\,dH(x|s^*) = \int_b^c \int_b^x \frac{G(z,s^*)\phi_\mu}{\rho+\phi}\,dz\,dH(x|s^*) \\
&= \int_b^c \frac{\rho}{(\rho+\phi)^2}(1-H(x|s^*))\phi_\mu\,dx
\end{aligned}
\tag{4.24}
$$

where $J(x,s^*)$ is given by equation (4.23),and the second line above used Fubini's theorem to change the order of integration. Similar to the planner's problem, the equilibrium condition of the dynamic model, equation (4.24), differs from the equilibrium condition of the static model, equation (4.16), by a factor $\rho/(\rho+\phi)^2$.

Proposition 22. *There is a unique equilibrium in the decentralized market.*

1. *If $\phi_\lambda = 0$, i.e., with invariant meeting technologies, firm entry is efficient.*
2. *If $\phi_\lambda < 0$ (> 0), in equilibrium there will be too much (little) firm entry.*

Proof. The proof is similar to that of Proposition 20.

Concerning uniqueness of the equilibrium: The decentralized equilibrium is characterized by equation (4.24). For x such that $1-H(x|s^*) > 0$, $1-H(x|s^*)$ is independent of s^*, and $\rho/(\rho+\phi)^2$ and $\phi_\mu(\lambda(s^*)(1-F(x,s^*)),\lambda(s^*))$ are both increasing in s^* because of Lemma 17 and Lemma 18, respectively. Therefore, the right hand of equation (4.24) is increasing in s^* and there is a unique solution for the equilibrium.

If the meeting technology is invariant ($\phi_\lambda = 0$), then the optimality condition for the social planner, equation (4.19), and the zero profit condition for the decentralized equilibrium, equation (4.24), coincide. It follows that firm entry is efficient.

Suppose $\phi_\lambda < 0$. Since the right hand side of equation (4.24) is increasing in s^*, comparing equations (4.19) and (4.24) implies there will be excessive firm entry. The same argument applies to the case $\phi_\lambda > 0$. □

4.4 Conclusion

In this chapter I analyze the efficiency of firm entry in a random search model. The model features general meeting technologies, on-the-job search, and information asymmetries. By using the new representation of meeting technologies introduced by Cai et al. (2015), (i) many of the existing results on entry efficiency can be encompassed in a unified framework, (ii) the chapter can define formally what a meeting externality is. I also introduced a mild new restriction (Assumption 2) on meeting technologies, and show that under this restriction the decentralized equilibrium is unique and that a negative (positive) externality in the meeting process results in too much (little) firm entry. Finally, by assuming that firms observe noisy signals about their productivities before entry (Assumption 3), it captures the McAfee (1993) case of ex ante heterogeneity and the Wolinsky (1988) case of ex post heterogeneity as special cases.

Appendix

Wage posting. Here I show that in both the static and the dynamic model, wage posting is an efficient mechanism. We start with a trivial case: if in the static model $P_0(\lambda) + P_1(\lambda) = 1$, then a worker can at most meet one firm and he will work for the firm with wage b. For other cases, I will show that a firm with a higher productivity will always post a higher wage. In the static model, denote the acceptance probability of wage w by $P_a(w)$. In the dynamic model, since the duration of a job depends only on the wage in the wage posting model, denote the (discounted) duration of a job by $P_a(w)$. Therefore, in both the static and the dynamic model, the present value of the firm is simply $(x - w)P_a(w)$, where $P_a(w)$ is a non-decreasing function of w.

For two firms with productivity x_1 and x_2, $x_1 < x_2$, suppose wages w_1 and w_2 are among the optimal choices for the two firms, respectively. Then by the incentive compatibility constraint,

$$(x_1 - w_1)P_a(w_1) \geq (x_1 - w_2)P_a(w_2)$$
$$(x_2 - w_2)P_a(w_2) \geq (x_2 - w_1)P_a(w_1)$$

Multiply the above two inequalities gives $(x_1 - w_1)(x_2 - w_2)P_a(w_1)P_a(w_2) \geq (x_1 - w_2)(x_2 - w_1)P_a(w_1)P_a(w_2)$. Therefore,

$$(x_2 - x_1)(w_2 - w_1) \geq 0,$$

which implies $w_2 \geq w_1$. As argued by Burdett and Judd (1983) and Burdett and Mortensen (1998), there must be no mass point in the wage distribution, and the probability that two firms with different productivity posting the same wage must be zero. Therefore, the wage posting mechanism is efficient.

Proof of Lemma 16. Suppose that for some λ_0, $P_0(\lambda_0)+P_1(\lambda_0)=1$. Thus with market tightness λ_0, a worker always meets at most one firm. Now consider a subgroup of firms with measure μ_0, because of invariance, we have that for any μ_0 with $0 \leq \mu_0 \leq \lambda_0$,

$$1 - P_0(\mu_0) = \frac{\mu_0}{\lambda_0} P_1(\lambda_0), \tag{.25}$$

where the left hand side is the probability that a worker meets at least one firm when market tightness is μ_0, and the right hand side is the probability that a worker meets at least one firm from the μ_0 subgroup when market tightness is λ_0. As a result, for $0 \leq \mu_0 \leq \lambda_0$, P_0 is a linear function and $P_0''(\mu_0) = 0$. Similarly consider any $\lambda > \lambda_0$,

$$1 - P_0(\mu_0) = \phi(\mu_0, \lambda)$$

Therefore, $0 = P_0''(\mu_0) = \phi_{\mu\mu}(\mu_0, \lambda)$. Note

$$\phi_{\mu\mu}(\mu, \lambda) = -\sum_{0}^{\infty} \frac{P_{n+2}(\lambda)}{\lambda^2}(1-\frac{\mu}{\lambda})^n, \tag{.26}$$

which implies $P_n(\lambda) = 0$ for $\lambda \geq \lambda_0$ and $n \geq 2$. Therefore $P_0(\lambda) + P_1(\lambda) = 1$ for all $\lambda \geq \lambda_0$. By equation (.25), P_0 is a linear function. Since P_0 is bounded, a contradiction arises and for any $\lambda > 0$, $P_0(\lambda) + P_1(\lambda) < 1$. Therefore, by equation (.26), $\phi(\mu, \lambda)$ is strictly concave in μ.

Verification of examples. Since the urn-ball meeting technology is invariant, it satisfies assumptions 1 and 2. Here I will verify the rest of the examples.

For the bilateral meeting technology: Since $\phi(\mu,\lambda) = P_1(\lambda)\mu/\lambda$, $\phi_\lambda(\mu,\lambda) = \mu\frac{P_1'(\lambda)\lambda - P_1(\lambda)}{\lambda^2}$. Since $P_1(\lambda)$ is strictly concave and $P_1(0) = 0$, $P_1'(\lambda) < \frac{P_1(\lambda)}{\lambda}$ and $\phi_\lambda(\mu,\lambda) < 0$. For Assumption 1, $\frac{d}{d\lambda}\phi(\lambda(1-z),\lambda) = P_1'(\lambda)(1-z) > 0$. For Assumption 2, $\frac{d}{d\lambda}\phi_\mu(\lambda(1-z),\lambda) = \frac{d}{d\lambda}\frac{P_1(\lambda)}{\lambda} = \frac{P_1'(\lambda)\lambda - P_1(\lambda)}{\lambda^2}$. As argued above, $P_1'(\lambda)\lambda - P_1(\lambda) < 0$. Thus Assumption 2 is satisfied.

Cobb-Douglass: Since $P_n(\lambda) = e^{-\lambda^\alpha}\frac{(\lambda^\alpha)^n}{n!}$, $\phi(\mu,\lambda) = 1-\sum_0^\infty e^{-\lambda^\alpha}\frac{(\lambda^\alpha)^n}{n!}(1-\frac{\mu}{\lambda})^n = 1-\sum_0^\infty e^{-\lambda^\alpha}\frac{(\lambda^\alpha-\mu\lambda^{\alpha-1})^n}{n!} = 1-e^{-\mu\lambda^{\alpha-1}}$. Therefore, $\phi_\lambda(\mu,\lambda) = -(1-\alpha)\mu\lambda^{\alpha-2}e^{-\mu\lambda^{\alpha-1}} < 0$. For Assumption 1, $\frac{d}{d\lambda}\phi(\lambda(1-z),\lambda) = \alpha(1-z)\lambda^{\alpha-1}e^{-\mu\lambda^{\alpha-1}} > 0$. For Assumption 2, $\frac{d}{d\lambda}\phi_\mu(\lambda(1-z),\lambda) = \frac{d}{d\lambda}\lambda^{\alpha-1}e^{-(1-z)\lambda^\alpha}$. Both $\lambda^{\alpha-1}$ and $e^{-(1-z)\lambda^\alpha}$ are decreasing in λ, thus $\frac{d}{d\lambda}\phi_\mu(\lambda(1-z),\lambda) < 0$ and Assumption 2 is satisfied.

Mixture between urn-ball and pairwise urn-ball: First we consider submarket B. As showed by Lester et al. (2015) and Cai et al. (2015), $\phi^B(\mu,\lambda) = 1-\sum_{n=0}^\infty e^{-\frac{\lambda}{2}}\frac{(\lambda/2)^n}{n!}(1-\frac{\mu}{\lambda})^{2n} = 1-\sum_0^\infty e^{-\frac{\lambda}{2}}\frac{(\frac{\lambda}{2}(1-\frac{\mu}{\lambda})^2)^n}{n!} = 1-\sum_0^\infty e^{-\frac{\lambda}{2}}\frac{(\frac{\lambda}{2}-\mu+\frac{\mu^2}{2\lambda})^n}{n!} = 1-e^{-\mu(1-\frac{\mu}{2\lambda})}$. Submarket A has the urn-ball meeting technology, thus $\phi^A(\mu,\lambda) = 1-e^{-\mu}$. Therefore, in the aggregate matching market $\phi(\mu,\lambda) = \zeta\phi^A(\mu,\lambda)+(1-\zeta)\phi^B(\mu,\lambda) = 1-\zeta e^{-\mu}-(1-\zeta)e^{-\mu(1-\frac{\mu}{2\lambda})}$. $\phi_\lambda(\mu,\lambda) = (1-\zeta)\frac{\mu^2}{2\lambda^2}e^{-\mu(1-\frac{\mu}{2\lambda})} > 0$. For Assumption 1, $\frac{d}{d\lambda}\phi(\lambda(1-z),\lambda) = (1-z)\zeta e^{-\lambda(1-z)}+(1-\zeta)\frac{1-z^2}{2}e^{-\lambda\frac{1-z^2}{2}} > 0$. For Assumption 2, $\frac{d}{d\lambda}\phi_\mu(\lambda(1-z),\lambda) = \frac{d}{d\lambda}\left(\zeta e^{-\lambda(1-z)}+(1-\zeta)ze^{-\lambda\frac{1-z^2}{2}}\right) < 0$. Therefore, Assumption 2 is also satisfied.

Derivations of equations (4.4), (4.5), and (4.7). For equation (4.4), we have

$$\begin{aligned} S &= b+\sum_{n=1}^\infty P_n(\lambda)\int_b^c (z-b)dF^n(z) = b+\sum_{n=1}^\infty P_n(\lambda)\int_b^c 1-F^n(z)dz \\ &= b+\int_b^c\sum_{n=1}^\infty P_n(\lambda)\left(1-F^n(z)\right)dz = b+\int_b^c \phi(\lambda(1-F(z)),\lambda)dz, \end{aligned}$$

where in the second equality in the first line we used integration by parts, in the first equality in the second line we interchange integration and summation, and in the second equality we used the definition of ϕ: $\phi(\lambda(1-F(z)),\lambda) = 1-\sum_{n=0}^\infty P_n(\lambda)F(z)^n = \sum_{n=0}^\infty P_n(\lambda)-\sum_{n=0}^\infty P_n(\lambda)F(z)^n = \sum_{n=1}^\infty P_n(\lambda)(1-F(z)^n)$.

A heuristic derivation of equation (4.5), the marginal contribution to surplus of a firm with productivity x, is as follows. Suppose we increase the measure of firms with productivity x by Δ, then in equation (4.4), the second argument of $\phi(\lambda(1-F(z)),\lambda)$ will increase by Δ, and the first argument $\lambda(1-F(z))$ will increase Δ if and only if $z \le x$. Therefore,

$$T(x) = \int_b^c \phi_\lambda(\lambda(1-F(z)),\lambda)dz+\int_b^x \phi_\mu(\lambda(1-F(z)),\lambda)dz.$$

Alternatively, we can make the above heuristic argument precise by discretizing the distribution $F(z)$ first. Suppose that there are n buyer types with $x_1 < x_2 <$

$\cdots < x_n$, with total measure $m_1, m_2, \ldots, m_n$. Then by equation (4.4), the total output can be written as

$$S = b + \sum_{k=1}^{n} (x_k - x_{k-1})\phi(M_k, M_1)$$

where we have defined $x_0 = b$ and $M_k = m_k + \cdots + m_n$ with $k = 1, \ldots, n$. The marginal contribution to total output by a firm with productivity x_i is thus,

$$\begin{aligned}\frac{\partial S}{\partial m_i} &= \sum_{k=1}^{n} (x_k - x_{k-1}) \left(\frac{dM_k}{dm_i} \phi_\mu(M_k, M_1) + \frac{dM_1}{dm_i} \phi_\lambda(M_k, M_1) \right) \\ &= \sum_{k=1}^{i} (x_k - x_{k-1}) \phi_\mu(M_k, M_1) + \sum_{k=1}^{n} (x_k - x_{k-1}) \phi_\lambda(M_k, M_1),\end{aligned}$$

where we have used the fact that $dM_k/dm_i = 1$ if $k \le i$ and $dM_k/dm_i = 0$ if $k > i$. Apparently the above equation will converge to equation (4.5) if discrete distribution converges to the productivity distribution F.

For equation (4.7): since $Q_n(\lambda)$ and $J_n(x)$ are the probability and the expected value of a firm with productivity x in an n-to-1 meeting,

$$\begin{aligned}J(x) &= \sum_{n=1}^{\infty} Q_n(\lambda) J_n(x) = \sum_{n=1}^{\infty} Q_n(\lambda) J_n(b) + \int_b^x \sum_{n=1}^{\infty} Q_n(\lambda) F^{n-1}(z) dz \\ &= J(b) + \int_b^x \phi_\mu(\lambda(1 - F(z)), \lambda) dz = \int_b^x \phi_\mu(\lambda(1 - F(z)), \lambda) dz,\end{aligned}$$

where in the second equality we used equation (4.6) to substitue out $J_n(x)$, in the first equality in the second line we used the definition of $J(b)$ and equation (4.2), and the second equality follows because we assume the wage mechanism is such that $J(b) = 0$.

CHAPTER 5

Minimum prices in a model with search frictions and price posting[66]

5.1 Introduction

In professional sports, collective bargaining agreements usually limit the maximum wage that an individual player can get or the total salary that a team can pay. One clear example is the market for football players in the UK from 1901 to 1961. During that period, the weekly maximum wage was the same for all players. Other examples include the current salary rules of the National Basketball Association (NBA) or the National Football League (NFL). In areas other than professional sports, a statutory maximum wage is not as common as a statutory minimum wage, but collective bargaining agreements, which are common in European countries, often specify the highest possible wage in a sector.

A minimum price in the product market is similar to a maximum wage in the labor market because both policies constrain the payoffs of buyers (workers). For example, Fairtrade International is an organization which sets minimum prices for a large range of agricultural products. In order to use the

[66] This chapter is based on Cai (2015b)

"Fair Trade" brand, importers or retailers have to pay at least the minimum price.[67] Similarly, a statutory minimum price for dairy products and books is common in many countries.

In this chapter, I study the effects of a statutory maximum wage/minimum price on the market wage/price distribution. Point of departure are the wage/price posting models with search frictions of Butters (1977), Chapter 1 of Mortensen (2003), and Burdett and Judd (1983), where the former two can be seen as a special case of the last. The effects of a maximum wage in a labor market with wage posting can always be restated in terms of effects of a minimum price in a product market with price posting. In this chapter, I mainly discuss the product market so that my results can be easily compared with the results of Burdett and Judd (1983).

In that model, price dispersion is a natural result because buyers can receive multiple offers and sellers face a positive probability to compete with each other. There cannot be a mass point in the price distribution because either there exists a small profitable downward deviation or a profitable upward deviation (at the competitive price) that gives a discrete jump in a seller's payoffs. Fershtman and Fishman (1994) show that a minimum wage or a maximum price will only change the support of the price distribution and pure price dispersion is preserved in equilibrium. In this chapter, I show that with a sufficiently high statutory minimum price, the price distribution will be degenerate at the minimum price.[68] For lower minimum prices, the price distribution will have a mass point at the minimum price and a gap around it in the support of the distribution. The reason why the mass point exists is that at a (binding) minimum price, small downward deviations are not possible. Furthermore, it is not attractive to offer a price just above the mass point because the price is almost the same as at the mass point while the selling probability is substantially lower.

[67] More detailed description can be found in Dragusanu et al. (2014).

[68] In the nonsequential search version of Burdett and Judd (1983), I will show that with a sufficiently high minimum price, the price distribution will become degenerate at the monopoly price.

5.2 Model

The basic model builds on Butters (1977), Chapter 1 of Mortensen (2003), and Burdett and Judd (1983). There are n_s sellers and n_b buyers. Each buyer has a unit demand for the indivisible good. Each seller sends out one advertisement containing a price quotation, and meets at most one buyer. The meeting process is constant-returns-to-scale, and the probability that a given buyer meets n sellers is exogenously given by $q_n(\theta)$ with $n = 0, 1, \ldots$, as in Burdett and Judd (1983), Eeckhout and Kircher (2010b), and Lester et al. (2015). As a result, the probability that a seller successfully meets a buyer who has in total n sellers is $s_n(\theta) = nq_n(\theta)/\theta$, where $n \geq 1$.[69] The probability of a seller not meeting any buyer is thus given by $s_0 = 1 - \sum_1^\infty s_k$. Following Burdett and Judd (1983), to make sure there will be equilibrium price dispersion we assume $q_0(\theta) + q_1(\theta) < 1$ for any $\theta > 0$, i.e. each buyer has a positive probability of meeting at least two sellers.

First consider the case without a statutory minimum price. As shown in Burdett and Judd (1983), the equilibrium distribution of market prices, $F(p)$, is dispersed without any mass point, i.e., there is pure price dispersion. The expected profit for a seller with price p is

$$\pi(p) = p \sum_{k=1}^{\infty} s_k (1 - F(p))^{k-1}, \tag{5.1}$$

where $(1 - F(p))^{k-1}$ is the probability that all other $k-1$ offers the contacted buyer have is higher than p.

Note that in equilibrium the highest market price must be p^*, the buyers' valuation. For a seller posting the highest price, the only winning scenario is the one where the contacted buyer has received no other price quote. In that scenario, the best option for the seller is to charge p^*. Finally, since all sellers are homogeneous, all posted prices should generate the same expected profit in equilibrium. So $\pi(p) = \pi(p^*)$ for any market price p, which implicitly defines a unique equilibrium distribution of prices $F(p)$. The lowest market price $\underline{p}$ is such that $F(\underline{p}) = 0$, and Equation (5.1) implies $\underline{p} = p^* s_1/(1 - s_0)$.

[69]We implicitly assumed that $\sum_1^\infty nq_n(\theta) \leq \theta$, since $\sum_1^\infty s_k(\theta) \leq 1$. The same assumption is adopted by Eeckhout and Kircher (2010b) and Lester et al. (2015).

Suppose now that the government imposes a statutory minimum price a. To make the minimum price binding, I assume $\underline{p} < a < p^*$. The only possible mass point in the new equilibrium price distribution must be the minimum price a, because a mass point at any other price will be subject to a profitable downward deviation. Since the equilibrium with pure price dispersion is not viable, in the new equilibrium there must be a mass point at the minimum price. Second, there will be a gap around the minimum price. With competition from sellers posting the minimum price, posting a slightly higher price will have a discrete drop in the selling probability

If there is a percentage x of sellers posting the minimum price a, then the expected seller profit with price a is

$$\begin{aligned} m(a,x) &= a\sum_{k=1}^{\infty} s_k \left(\sum_{i=1}^{k} \frac{(k-1)!}{(i-1)!(k-i)!} x^{i-1}(1-x)^{k-i}\frac{1}{i} \right) \\ &= \frac{a}{\theta}\sum_{k=1}^{\infty} q_k \frac{1-(1-x)^k}{x}, \end{aligned} \tag{5.2}$$

where in the first line $s_k = kq_k/\theta$ is the probability that the seller meets a buyer with k sellers, and the term in parenthesis is the probability that conditional on that the buyer has k sellers, the buyer has i sellers offering the minimum price a, and the seller gets chosen by the buyer.

Lemma 23. *$m(a,x)$ is strictly decreasing in x.*

Proof. By Equation (5.2), $m(a,x) = \frac{a}{\theta}\sum_{k=1}^{\infty} q_k \frac{1-(1-x)^k}{x}$. We will prove for each $k \geq 2$, $(1-(1-x)^k)/x$ is strictly decreasing in x. Of course when $k=1$, it is a constant.

First we need the following fact: for $0 < x \leq 1$ and $k \geq 1$,

$$(1-x)^k < \frac{1}{1+kx}, \tag{5.3}$$

which can easily be proved by induction.

The derivative of $(1-(1-x)^k)/x$ can be written as $\frac{(1-x)^{k-1}(1+(k-1)x)-1}{x^2}$, which is zero if $k=1$ and strictly negative if $k \geq 2$ by Equation (5.3). Since $\sum_2^{\infty} q_k > 0$, $\frac{\partial m(a,x)}{\partial x} < 0$. $\square$

Proposition 24. *If the minimum price a is sufficiently high,*

$$a \geq \frac{q_1}{1-q_0} p^*,$$

then all sellers will post the minimum price.

If the minimum price a is modestly high,

$$\underline{p} < a < \frac{q_1}{1-q_0} p^*,$$

where $\underline{p} = p^* s_1/(1-s_0)$ is the lowest market price in the equilibrium without any price control, then there is a unique equilibrium price distribution with a mass point at the the minimum price a and price dispersion on an interval $[b, p^*]$ with $a < b$. In this case, the expected value for sellers or buyers is the same as that without the minimum price.

Proof. First note that if there are prices other than a prevailing in equilibrium, then the highest market price must be p^*.

If $a \geq p^* q_1/(1-q_0)$, then Equations (5.1) and (5.2) imply $m(a,1) \geq \pi(p^*)$. Furthermore, Lemma 23 implies for any x with $0 \leq x < 1$, $m(a,x) > m(a,1) \geq \pi(p^*)$. Thus the only possible equilibrium is that all sellers post the monopoly price p^*.

If $\underline{p} < a < p^* q_1/(1-q_0)$, by Lemma 23 there must be a unique x such that $m(a,x) = \pi(p^*)$ with $0 < x < 1$. Thus not all buyers will post the minimum price, and the highest market price must be p^*. For prices $p > b$, the equilibrium price distribution is again implicitly determined by Equation (5.1), and b is such that $F(b) = x$. A seller's expected profit is always $\pi(p^*)$ for $\underline{p} < a < p^* q_1/(1-q_0)$, which is the same as without the minimum price. Since the total surplus of sellers and buyers is determined by the number of trades, which is not affected the minimum price, the expected value of buyers also stays the same. □

Connection to Burdett and Judd (1983)

In the above model, the meeting technology is exogenous. Burdett and Judd (1983) consider two extensions: (i) noisy search and (ii) nonsequential search. In the following, I show that these extensions do not change my main conclusion. The only exception is that in the nonsequential search model, for a sufficiently high statutory minimum price, instead of posting the minimum price, all sellers will post the monopoly price.

Noisy search. In the noisy search model of Burdett and Judd (1983), buyers pay c to receive an unknown number of price offers. As before, the probability of n offers is exogenously given and denoted by q_n.[70] Buyers are allowed to continue to sample price offers if they want to. After paying c and receiving n (random) price offers, buyers will use the following reservation price strategy: they will accept the lowest price offer if and only if it is lower than a reservation price, z. Otherwise, they decline all price offers and continue to search. As argued in Burdett and Judd (1983), knowing that buyers use the reservation price strategy, sellers will never post a price higher than z. If they do, they will never be able to sell their product. As a result, in equilibrium all sellers will post prices below z and buyers only need to search once.

If $z = p^*$, then of course we are back to the previous model. In this case, after introducing a statutory minimum price, there will be a mass point at a and the price dispersion will be reduced. As a result, buyers' gain from resampling will not increase, and the reservation price will stay at p^*. If $z < p^*$, as long as the minimum price is modestly high, by Proposition 24 the gain from resampling will be the same, and the reservation price z will stay the same. If the minimum price is high enough, then the price distribution will become degenerate at a, and the reservation price $z = a + c$.

Nonsequential search. In the nonsequential search model of Burdett and Judd (1983), buyers can choose how many price offers, say n, they want to have, but buyers must pay c for each price quotation. Contrary to the monopoly price equilibrium which always exists (see Diamond (1971)), Burdett and Judd (1983) argue that there exist pure price dispersion equilibria, where buyers randomize between one and two price quotations with probability q and $1 - q$ respectively. The expected payoffs of one and two price offers for buyers are the same in equilibrium. Burdett and Judd (1983) show that there exists a threshold

[70]Burdett and Judd (1983) assume $q_0 = 0$. Since this restriction doesn't matter, I will continue to use the general setup where q_0 is not restricted.

c^* such that for any $c < c^*$ two equilibria exist, with different q's. Suppose the lowest market prices for the two equilibria are $\underline{p}_1$ and $\underline{p}_2$, with corresponding q_1 and q_2, $q_1 < q_2$. Since in Equilibrium 1, more buyers will receive two price quotations, the competition among sellers is stronger and $\underline{p}_1 < \underline{p}_2$.

Now consider the introduction of a statutory minimum price a. If a is slightly higher than $\underline{p}_1$, then the price distribution of Equilibrium 1 is no longer sustainable and the new price distribution of Equilibrium 1 will have a mass point at a by Proposition 24, but the price distribution of Equilibrium 2 is unaffected since $a < \underline{p}_2$. If a is slightly higher than $\underline{p}_2$, then there will be no equilibrium with pure price dispersion. If a continues to increase, then more sellers will choose to post the minimum price. This will weaken buyers' incentives to search. As a result, buyers will search only once and all sellers will post the monopoly price in response.

5.3 Extensions: Heterogeneity and Efficiency

Heterogeneity. To make the model simple and easy to compare with the literature, I have assumed that both sellers and buyers are homogeneous. A natural question is, will the main conclusion continue to hold in the presence of heterogeneity? For example, sellers can have different production cost. The previous logic of the minimum price continues to hold in that case. As long as the statutory minimum price is above the lowest price in the market without price control, there will be a mass point at the minimum price. The reason is the same as before: The equilibrium with pure price dispersion will not exist anymore, and any mass point above the statutory minimum price is subject to profitable downward deviations. As a result, in equilibrium there must be a mass point at the minimum price.

Efficiency. With homogeneous buyers and sellers, if the number of sellers is fixed, then the price distribution does not affect the total surplus since the total number of contacts/trades is determined by an exogenous, frictional search process.[71] However, in the long run, market tightness should be endogenous. By assuming that sellers have a fixed entry cost, we can ask whether a statutory

[71] The number of trade per buyer is $1 - q_0(\theta)$, which doesn't depend on the wage distribution.

minimum price could increase social efficiency. Butters (1977) shows that with homogeneous buyers and sellers, entry in the decentralized market with the urn-ball meeting technology is already efficient, so there is no role for a statutory minimum price. However, Cai (2015a) shows that with other meeting technologies, the decentralized market can have too little entry.[72] An optimal statutory minimum price can then achieve the social optimum by increasing seller profits and thus seller entry.[73]

5.4 Final remarks

This chapter shows that price dispersion can disappear completely with the introduction of a minimum price. When the minimum price is modestly high, some sellers will post the minimum price and the other sellers will post prices away from the minimum price. Consequently, there will be a gap in the support of the market price distribution.

Burdett and Mortensen (1998) is a labor market version of Burdett and Judd (1983). In that model, workers can search for better jobs when they are already employed. On-the-job search rules out mass points in the wage distribution because firms could always offer a slightly higher wage, which would result in a discrete jump of the hiring rate. However, if there is a statutory maximum wage and the wage can't be increased further, then a mass point at the maximum wage is also possible in that setting. In the case of homogeneous firms and workers, if we assume that a worker does not leave the current firm when a poaching firm offers the same wage, the dispersed wage equilibrium is very sensitive to a statutory maximum wage. Any statutory maximum wage smaller

[72] One meeting technology featuring too little entry is the following: Suppose that with probabilities ζ and $1-\zeta$, $0<\zeta<1$, an agent (a seller or a buyer) will be allocated to submarkets A and B, respectively. In submarket A, sellers and buyers meet according to the urn-ball model. In submarket B, sellers and buyers meet according to the pairwise urn-ball model introduced by Lester et al. (2015). That is, in submarket B, sellers first need to form pairs, and then as a pair, two sellers will jointly meet a buyer randomly. Formally, in submarket A, $q_n(\lambda \mid A) = e^{-\lambda}\frac{\lambda^n}{n!}$ for $n = 0,1,2,\ldots$, and in submarket B, $q_n(\lambda \mid B) = e^{-\lambda/2}\frac{(\lambda/2)^n}{(n/2)!}$ for $n = 0,2,4,\ldots$ and $q_n(\lambda \mid B) = 0$ for $n = 1,3,5,\ldots$. See Cai (2015a) for details.

[73] Here I consider the efficiency only through seller entry. However, a more complete picture should also include search costs of buyers. Gautier et al. (2015) argue that price dispersion may make buyers collect too many price quotations (because of a rent seeking externality). Therefore reducing price dispersion can be welfare improving, which creates a role for a minimum price.

than the highest wage prevailing in the market without wage control will make all firms post the statutory maximum wage, since by switching to the statutory maximum wage, firms will not lose workers to competitors. The situation is very different for a statutory minimum wage. A minimum wage will only shift the support of the wage distribution, but the shape of the wage distribution will remain intact.

CHAPTER 6

Summary and conclusions

This thesis consists of four chapters on search theory. Chapter 2 is the most applied one. It studies collective wage bargaining in a search model with two sided heterogeneity and on-the-job search. This chapter compares collective bargaining agreements (CBA) with a decentralized bargaining outcome case. Under CBA, a union chooses a pay-scale schedule and the firm can select a wage from this pay scale after observing match quality. An advantage of collective bargaining agreements is that search and business-stealing externalities can be internalized, since firms do not internalize the output loss of the firms they poach a worker from. Absent CBA, output can be excessive. A disadvantage of CBA is that it takes more time before an optimal allocation is reached. Under the decentralized bargaining, a worker always receive a fixed share (β) of the match surplus, which is defined as the additional value generated by the match compared to the worker being unemployed and the job being vacant. What the most desirable system is, depends on the worker's bargaining power (β) and the relative efficiency of on- versus off- the job search. Job flow data from the Netherlands and the US are used to infer the empirical values of the relative efficiency of on- versus off- the job search, and find for both countries that as long as β lies between 0.1 and 0.7, CBA is less desirable.

Chapter 3 considers a market in which sellers compete for heterogeneous buyers by posting selling mechanisms. Buyers can observe all posted mechanisms before deciding where to visit. The number of buyers that visit a seller depend on the the queue length in a stochastic way, which is referred to as the meeting technology. This chapter analyzes how the properties of the meeting technology affect the sorting of buyers across sellers and the posted mechanisms. A new function, $\phi(\mu, \lambda)$, is introduced, which specifies the probability for a seller to meet at least one buyer from a given subset, where μ is the relative measure of buyers in the given subset and λ is the relative measure of all buyers. This new function is a one-to-one transformation of the meeting technology and helps to clarify and extend many of the existing results in competing auctions. This chapter shows that: (i) a separate submarket for each type of buyer is the efficient outcome if and only if meetings are bilateral, i.e., $\phi(\mu, \lambda)$ is linear in μ, (ii) a single market with all buyer types is the efficient outcome if and only if $\phi(\mu, \lambda)$ is concave in (μ, λ). Both outcomes can be decentralized by sellers posting auctions combined with an appropriate fee or subsidy. The fee will internalize the externalities imposed by buyers on each other. Finally, different classes of meeting technologies like invariance and non-rivalry are expressed in terms of ϕ and a Venn diagram shows how they relate to each other.

Chapter 4 considers the efficiency of firm entry in a model with on-the-job search where firms have private information on productivity. In each period before entry, firms observe a signal indicating potential productivity (this allows me to treat ex post and ex ante heterogeneity as special cases). Firms will only enter the market if their private signals are higher than a threshold. Both unemployed and employed workers receive job offers according to a meeting technology. Unlike Chapter 3, firms do not in general ex ante observe the terms of trade; they learn it after meeting a worker. Thus search is random. This chapter shows that whether the decentralized equilibrium is efficient depends on the meeting technology. If additional firm entry does not affect the meeting probabilities of existing firms (as under the urn ball), then the search equilibrium is efficient; if it decreases (increases) the meeting probabilities of existing firms, then in equilibrium there will be too much (little) firm entry. I consider not only wage posting but all wage mechanisms that have the following properties: i) in the absence of switching costs, workers will always move to the more productive firms, ii) the firm with productivity equal to the workers' value of leisure (the lowest productive firm) receives zero.

Chapter 5 considers the effects of a statutory minimum price in a random search model with price posting. Some countries apply this for gasoline, food, or books. In the price posting model, sellers post and commit to the posted price. If buyers can receive two or more price offers at the same time, they choose the lowest price or in case of a tie-break, they randomize. Thus sellers face a positive probability to compete with each other. No seller will post a price equal to his or her reserve value because there is always some chance of selling at a higher price, which will result in a positive expected profit. Also, there can not exist mass point in the market price distribution because then a small profitable downward deviation is always possible. Thus there will be pure market price dispersion (Burdett and Judd (1983)). I show that with a minimum price, the price distribution will have a mass point. If the minimum price is sufficiently high, the price distribution will become degenerate. The reason why the mass point exists is that at a (binding) minimum price, small downward deviations are no longer possible. Furthermore, it is not attractive to offer a price just above the mass point because the selling price is almost the same as at the minimum price while the selling probability is substantially lower. Therefore, firms either post the minimum price or substantially higher prices. Consequently, there will be a gap in the market price distribution.

Samenvatting (Summary in Dutch)

Dit proefschrift bestaat uit vier hoofdstukken over zoektheorie.

Hoofdstuk 2 onderzoekt hoe CAO loononderhandelingen van invloed zijn op sociale efficiëntie. In veel Europese landen wordt de verzameling van mogelijke lonen begrensd door CAO loononderhandelingen. Hoofdstuk 2 richt zich op de volgende afweging. Onder een CAO wordt de spreiding in lonen verkleind, zodat een bedrijf met een hoge productiviteit hetzelfde loon kan bieden als een minder productief bedrijf. Dus zelfs als een werknemer van het minder productieve bedrijf benaderd wordt door het productievere bedrijf, kan het gebeuren dat de werknemer niet van baan verandert en dit leidt tot een lagere gemiddelde productiviteit in de economie. Aan de andere kant, in een gedecentraliseerd evenwicht zonder CAO internaliseren bedrijven de zoek- en *business stealing* externaliteiten die zij elkaar opleggen niet, waardoor een overschot aan vacatures wordt gecreëerd. Het netto-effect hangt af van het relatieve gewicht van de zojuist beschreven afwegingen. Met gedecentraliseerde onderhandelingen ontvangt een werknemer altijd een vast aandeel (β) van het *match surplus*, wat gedefinieerd is als de toegevoegde waarde van de overeenkomst vergeleken met de situatie waar de werknemer werkloos is en de baan onbezet is. Welke van de twee systemen het meest wenselijk is, hangt af van de onderhandelingsmacht van de werknemer (β) en de relatieve efficiëntie van zoeken met of zonder baan. Gegevens over baanstromen van Nederland en de Verenigde Staten zijn gebruikt om de empirische waarden van de relatieve efficiëntie van met of zonder baan solliciteren af te leiden, en voor beide landen wordt gevonden dat een CAO minder wenselijk is als β tussen de 0.1 en 0.7 ligt.

Hoofdstuk 3 bekijkt een markt waarin verkopers met elkaar concurreren om heterogene kopers door het plaatsen van verkoop mechanismes. Kopers observeren alle geplaatste mechanismes voordat ze besluiten welke verkoper ze gaan bezoeken. Het aantal kopers dat een verkoper bezoekt, hangt af van de lengte van de rij op een stochastische wijze, welke wordt aangeduid als ontmoetingstechnologie. Dit hoofdstuk onderzoekt hoe de eigenschappen van de ontmoetingstechnologie de rangschikking van kopers over de verkopers en geplaatste mechanismes beïnvloedt. Een nieuwe functie, $\phi(\mu,\lambda)$, wordt geïntroduceerd. Deze functie specificeert de kans dat een verkoper ten minste een koper uit een bepaalde deelverzameling ontmoet, waarbij μ het relatieve aantal van de kopers en λ het relatieve aantal van de verkopers in deze deelverzameling is. Deze nieuwe functie is een één-op-één omzetting van de ontmoetingstechnologie en kan veel van de bestaande resultaten in concurrerende veilingen duiden en aanvullen. Dit hoofdstuk laat zien dat: (i) een gescheiden submarkt voor elk type koper de efficiënte uitkomst is als en slechts dan als ontmoetingen bilateraal zijn, d.w.z., $\phi(\mu,\lambda)$ is lineair in μ, (ii) een interne markt met alle typen kopers is de efficiënte uitkomst als en slechts dan als $\phi(\mu,\lambda)$ concaaf is in (μ,λ) . Beide uitkomsten kunnen worden gedecentraliseerd door het plaatsen van veilingen door verkopers, gecombineerd met een passende vergoeding of subsidie. De vergoeding internaliseert de externaliteiten die de kopers veroorzaken. Ten slotte, verschillende klassen van ontmoetingstechnologiën zoals invariantie en non-rivaliteit worden uitgedrukt in ϕ en een Venn Diagram toont hoe zij zich tot elkaar verhouden.

Hoofdstuk 4 beschouwt de efficiëntie van toetredingen van bedrijven in een model met *on-the-job search* waarbij bedrijven eigen informatie hebben over productiviteit. In elke periode voor de toetreding, observeren bedrijven een signaal dat de potentiele productiviteit aangeeft (hierdoor kan ik ex-post en ex-ante heterogeniteit als speciale gevallen behandelen). Bedrijven treden toe tot de markt als hun eigen signalen hoger zijn dan een bepaalde grenswaarde. Zowel werklozen als werknemers ontvangen baanaanbiedingen volgens een ontmoetingstechnologie. In tegenstelling tot Hoofdstuk 3 observeren bedrijven in het algemeen de voorwaarden van de handel niet; ze vernemen dit pas na de ontmoeting met de werknemer. De zoektocht is dus willekeurig. Dit hoofdstuk laat zien dat het wel of niet efficiënt zijn van een gedecentraliseerd evenwicht, afhangt van de ontmoetingstechnologie. Als een extra toetreding van een bedrijf geen invloed heeft op de ontmoetingskansen van bestaande bedrijven (zoals in het vaasmodel), dan is het zoekevenwicht efficiënt; als het de ontmoetingskansen van bestaande bedrijven verlaagt (verhoogt), dan zijn er in het evenwicht te veel (weinig) toetredingen. Ik bekijk niet alleen loonzetting maar alle loonmechanismen met de volgende eigenschappen: i) In een situatie zonder overstapkosten gaan werknemers altijd naar de meer productieve bedrijven, ii) het bedrijf met een productiviteit gelijk aan de werknemers prijs van vrije tijd (het minst productieve bedrijf), ontvangt niets.

Hoofdstuk 5 bekijkt de effecten van een wettelijke minimumprijs in een willekeurig zoekmodel met prijsstelling. Sommige landen passen dit toe voor benzine, voedsel of boeken. In het model met prijsstelling, stellen verkopers de prijs vast en houden hierna aan deze prijs vast. Als kopers twee of meer aanbiedingen op hetzelfde moment ontvangen, kiezen ze de laagste prijs of, in het geval van gelijke prijzen, kiezen ze er een op basis van willekeur. Verkopers hebben dus een positieve kans om met elkaar te concurreren. Geen enkele verkoper zal een prijs kiezen die gelijk is aan zijn of haar reserveringswaarde, aangezien er altijd een kans is om te verkopen tegen een hogere prijs, wat een hogere verwachte winst zal opleveren. Bovendien kan er geen massapunt in de marktprijs verdeling bestaan omdat er dan altijd een kleine winstgevende neerwaartse beweging mogelijk is. Dus zal er pure spreiding zijn van de marktprijzen (Burdett en Judd (1983)). Ik laat zien dat in de situatie met een minimumprijs, er een massapunt in de verdeling van prijzen bestaat. Als de minimumprijs hoog genoeg is, concentreren de prijzen zich op één massapunt. De oorzaak van dit massapunt is dat op een (bindende) minimumprijs, kleine neerwaartse bewegingen niet meer mogelijk zijn. Bovendien is het niet aantrekkelijk om een prijs vlak boven het massapunt aan te bieden, omdat de verkoopprijs bijna gelijk is aan de minimumprijs terwijl de verkoopkans aanzienlijk kleiner is. Daarom stellen bedrijven hun prijzen vast op de minimumprijs of aanzienlijk hogere prijzen. Zodoende ontstaat er een kloof in de verdeling van marktprijzen.

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